THE WORKS OF Edmund Gunter.

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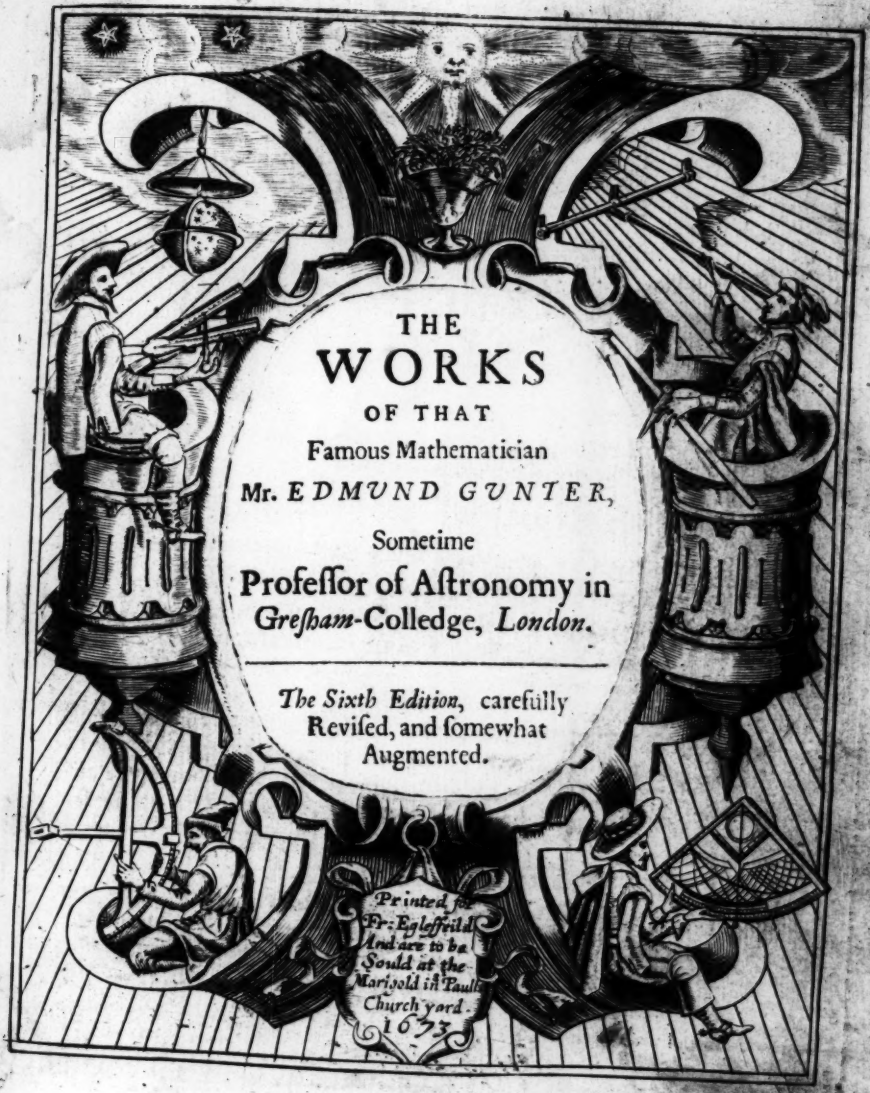


THE WORKS

OF THAT

Famous Mathematician

Mr. *EDMUND GUNTER*,

Sometime

Profeſſor of Aſtronomy in *Greſham*-Colledge, *London*.

The Sixth Edition, carefully Reviſed, and ſomewhat Augmented.

Printed for Fr: Egleſfield And are to be Sould at the Marigold in Paul's Church yard. 1673.

OXFORD

THE

WORKS

OF

EDMUND GUNTER:

Containing the *Deſcription* and *Uſe* of the

Sector, Croſs-ſtaff, Bow, Quadrant,

And other Inſtruments.

With a Canon of Artificial Sines and Tangents to a Radius of 10.00000 parts, and the Logarithms from an Unite to 10000:

The Uſes whereof are illuſtrated in the Practice of

Arithmetick, *Geometry*, *Aſtronomy*, *Navigation*, *Dialling*, and *Fortification*.

And ſome Queſtions in Navigation added by Mr. *Henry Bond*, Teacher of Mathematicks in *Rateliff*, near *London*.

To which is added,

The Deſcription and Uſe of another Sector and Quadrant, both of them invented by Mr. *Sam. Foſter*, Late Profeſſor of Aſtronomy in *Greſham* Colledge, *London*, furniſhed with more Lines, and differing from thoſe of Mr. *Gunters* both in form and manner of Working.

The Sixth Edition.

Diligently Corrected, and divers neceſſary Things and Matters (pertinent thereunto) added, throughout the whole Work, not before Printed.

By *William Leybourn*, Philomath.

LONDON:
Printed for *Francis Egleſfield* at the Marigold in St. *Paul's* Churchyard. MDCLXXX.

TO THE

RIGHT HONOURABLE,

JOHN,

Earl of *Bridgwater*, Viſcount *Brackley*, Baron *Elleſmore*: One of the Lords of His Majeſties moſt Honourable Privy Council, and Lord Lieutenant of the County of *Buckingham*.

My LORD,

THeſe *Works of the Learned* Gunter, *do naturally and of right, addreſs themſelves to your Honours Patronage: having been originally Inſignalized with the Titles of your Renowned Anceſtors, under whom (near Fifty years ſince) they received their firſt Life: So that they ſeem to be Intailed on your Illuſtrious Family: Eſpecially, conſidering the whole World owns your Lordſhip, no leſs Heir to your Anceſtors reſplendent Demeans, than of their love to Noble Arts, amongſt which thoſe of the Mathematicks, is none of the meaneſt you are Maſter of.*

My

The Epistle Dedicatory.

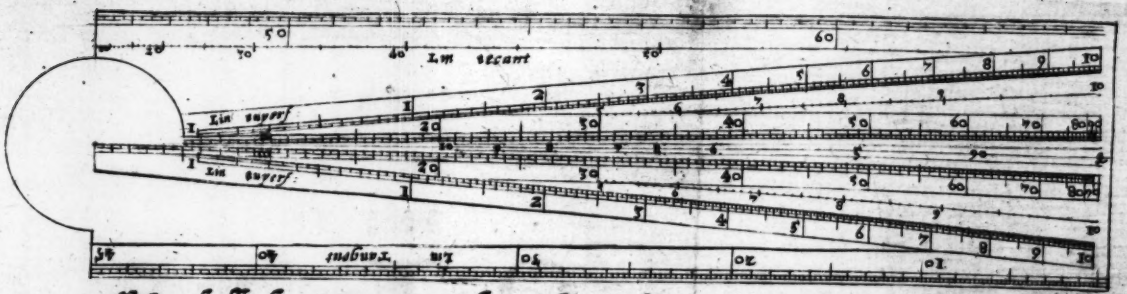
My Lord, The World taking example by so good a Commendum has encouraged this Work to this Sixth Edition. But it having met with the ill usage in former Impressions to have contracted several Typographical Sphalmata, which did somewhat disfigure its beauty, I thought it requisite to bestow some pains in their removal; as also to furnish it with the attendance of some compleat Tracts of Mr. Samuel Foster *our Learned Author's Successor in his Astronomical Profession in* Gresham Colledge : *What other Additions I have made to the Work, in several places, which are not only pertinent, but necessary, the Reader is acquainted with in the Preface. My Lord, This Work being arrived to this state of Perfection, pleads for a bolder access to your Honours hands; and makes it humbly confident to find your Honour no less favourable to it, now grown up, than your Predecessors have been to its infancy. My Lord, I have derived, likewise, hence some share of that humble confidence, that your Honour will pardon this presumption, of my subscribing my self,*

My Lord,

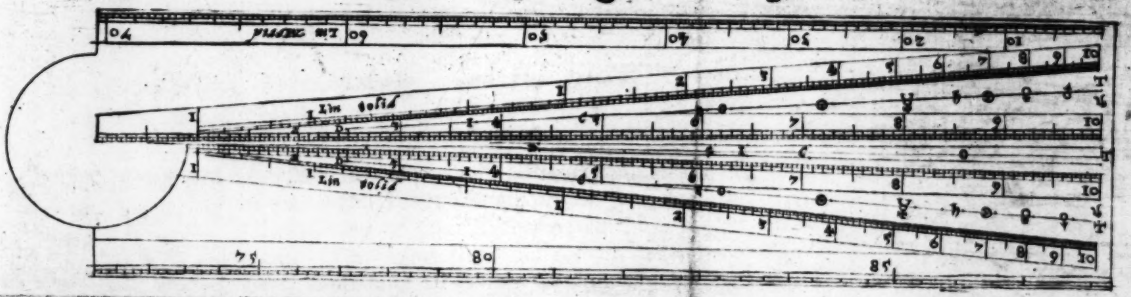
Your Lordships most obliged

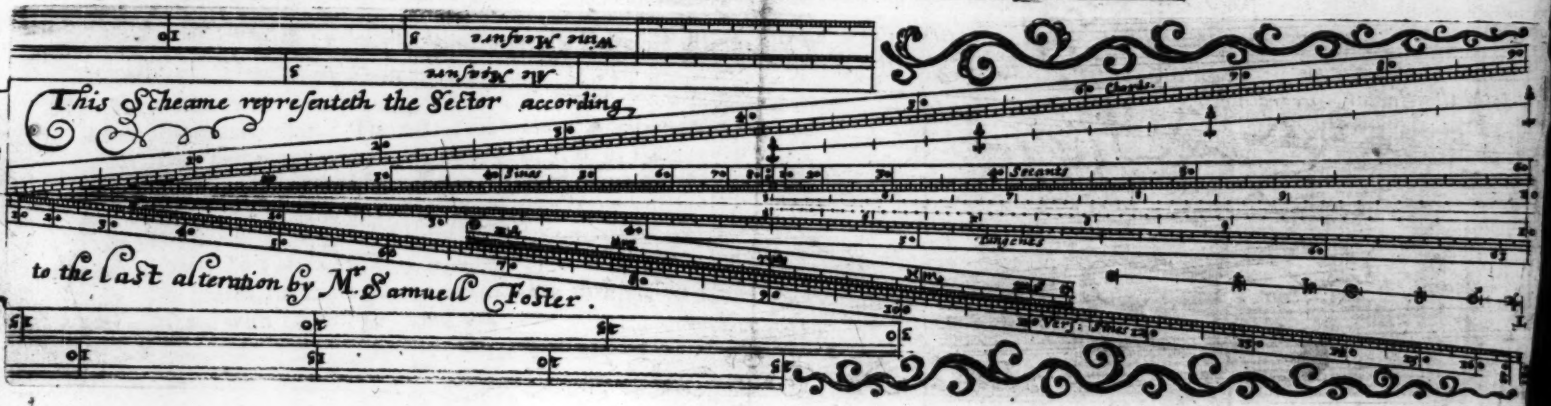
and obsequious Servant,

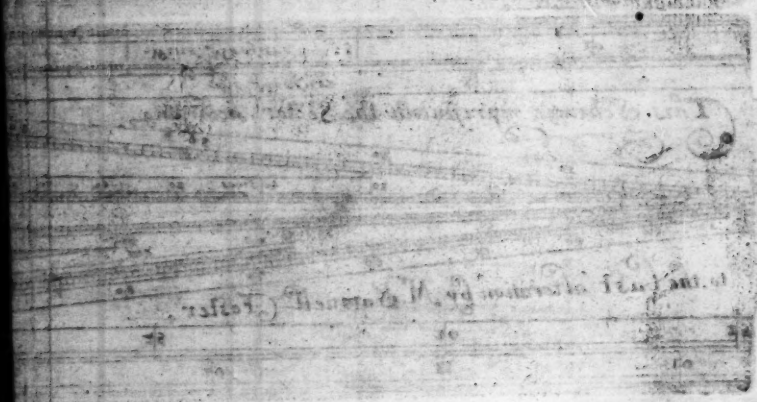
William Leybourn.



These and all other Instruments for the Mathematicall practice are made by Walter Hayes at the Croſse daggers in Moore ſeilds next dore to the Popes head Taverne and alſso by Anthony Thompſon in Hoſier Lane neare Smithſeild.







THE FIRST BOOK OF THE SECTOR.

CHAP. I.

The Description, the Making, and the General Use of the SECTOR.

A SECTOR in Geometry, is a Figure comprehended of two right Lines containing an Angle at the Center, and of the Circumference assumed by them. This Geometrical Instrument having two Legs, conteining all variety of Angles, and the distance of the Feet, representing the Subtenses of the Circumference, is therefore called by the same name.

It conteineth 12 several Lines or Scales, of which 7 are general, the other 5 more particular. The first is the Scale of Line, divided into 100 equal parts, and numbred by 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

The second, the Lines of Superficies, divided into 100 unequal parts, and numbred by 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

3. The third, the Lines of Solids, divided into 1000 unequal parts, and numbred by 1, 1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

4. The fourth, the Lines of Sines and Chords, divided into 90 degrees, and numbred with 10, 20, 30 unto 90.

These

Theſe four Lines, of *Lines*, of *Superficies*, of *Solids*, and of *Sines*, are all drawn from the Center of the *Sector* almoſt to the end of the Legs. They are drawn on both the Legs, that every Line may have his fellow. All of them are of one Length, that they may anſwer one to the other: And every one hath his Parallels, that the eye may the better diſtinguiſh the Diviſions. But of the Parallels thoſe only which are inward-moſt contein the true Diviſions.

There are three other general Lines, which becauſe they are infinite are placed on the Side of the Sector.

5. The firſt a Line of *Tangents*, numbred with 10, 20, 30, 40, 50, 60, ſignifying ſo many degrees from the beginning of the Line, of which 45 are equal to the whole Line of Sines, the reſt follow as the length of the Sector will bear.

6. The ſecond, a Line of Secants, divided by Pricks into 60 degrees, is the ſame with that of the Line of Tangents to which it is joyned.

7. The third is the Meridian Line, or Line of Rumbs, divided unequally into degrees, of which the firſt 70 are almoſt equal to the whole Line of Sines, the reſt follow unto 85, according to the Length of the Sector.

Of the particular Lines inſerted among the general, becauſe there was void ſpace.

8. The firſt are the Lines of Quadrature placed between the Lines of Sines, and noted with 10, 9, 8, 7, S, 6, 5, 90, Q.

9. The ſecond, the Lines of Segments, placed between the Lines of Sines and Superficies, divided into 50 parts, and numbred with 5, 6, 7, 8, 9, 10.

10. The third, the Lines of inſcribed bodies in the ſame Sphere, placed between the Scales of Lines, and noted with *D. S. I. C. O. T.*

11. The fourth, the Lines of Equated Bodies, placed between the Lines of Lines and Solids, and noted with *D. I. C. S. O. T.*

12. The fifth, are the Lines of Metals, inſerted with the Lines of Equated Bodies, (there being room ſufficient) and noted with the Characters ☉ ☿ ♄ ☽ ♀ ♂ ¥.

There remain the Edges of the Sector, and on the one I have ſet a Line of Inches, which are the twelfth parts of a Foot *Engliſh*: on the other a leſſer Line of Tangents to which the Gnomon is Radius.

2. *Of*

2. *Of the making of the* SECTOR.

LEt a Ruler be firſt made either of Braſs or of Wood like unto the Figure before the Book annexed. Which may open and ſhut upon his Center: the Head of it may be about the twelfth part of the whole Length, that it may bear the moveable Foot, and yet the moſt part of the Diviſions may fall without it. Then let a moveable Gnomon be ſet at the end of the moveable Foot, and there turn upon an Axis, ſo as it may ſometimes ſtand at a right Angle with the Feet; and ſometimes be incloſed within the Feet. But this is well known to the Work-man.

For drawing of the Lines. Upon the Center of the Sector, and Semidiameter ſomewhat ſhorter than one of the Feet, draw an occult Ark of a Circle, croſſing the Cloſure of the inward Edge of the Sector, about the Letter *T*.

In this Ark, at one degree on either Side from the Edge, draw right Lines from the Center, fitting them with Parallels, and divide them into an hundred equal parts, with Subdiviſions into 2, 5, or 10, as the Line will bear, but let the Numbers ſet to them, be only 1, 2, 3, 4. *&c.* unto 10, as in the Example. Theſe Lines ſo divided, I call the Lines or Scales of Lines; and they are the ground of all the reſt.

In this Ark, at 5 degrees on either Side, from the Edge near *T*, draw other right Lines from the Center, and fit them with Parallels: theſe ſhall ſerve for the Lines of Solids.

Then on the other Side of the Sector, in like manner, upon the Center, and equal Semidiameter, draw another like Ark of a Circle: and here again at one degree near on either Side from the Edge, near the letter *Q*, draw right Lines from the Center, and fit them with Parallels: Theſe ſhall ſerve for the Lines of Sines.

At 5 degrees, on either Side from the Edge near *Q*, draw other right Lines from the Center, and fit them with Parallels: theſe ſhall ſerve for the Lines of Superficies.

Theſe four principal Lines being drawn, and fitted with Parallels, we may draw other Lines in the middle between the Edges and the Lines of Lines, which ſhall ſerve for the Lines of Inſcribed Bodies, and others between the Edges and the Sines for the Lines of Quadrature. And ſo the reſt as in the Example.

3. *To divide the Lines of Superficies.*

SEeing the Superficies do hold in the Proportion of their *homologal* Sides duplicated by the 29. *Prop.* 6. *Lib. Euclid.* If you ſhall find mean Proportionals between the whole Side, and each hundred part of the like Side, by the 13. *Prop.* 6. *Lib. Euclid.* all of them cutting the ſame Line, that Line ſo cut ſhall contain the Diviſions required; wherefore upon the Center A, and Semidiameter equal to the Line of Lines, deſcribe a Semicircle A C B D, with A B perpendicular to the Diameter C D. And let the Semidiameter A D be divided as the Line of Lines into an hundred parts, and A E the one half of A C divided alſo into an hundred parts, ſo ſhall the Diviſions in A E be the Centers from whence you ſhall deſcribe the Semicircles C 10. C 20. C 30. *&c.* dividing the Line A B into an hundred unequal parts: and this Line A B ſo divided ſhall be the Line of Superficies, and muſt be transferred into the Sector. But let the numbers ſet to them be only 1. 1. 2. 3. unto 10. as in the Example.

Or theſe Lines of Superficies may otherwiſe be transferred into the Sector, out of the Line of Lines, by a Table of Square Roots; For the Root taken out of the Line of Lines, ſhall give the Square in the Lines of Superficies.

As, to inſcribe the Diviſion of 25 in the Lines of Superficies; put ſix Ciphers to 25, and make it 25000000, then find the Square Root of this Number, which will be 5000.

Take therefore 5000 out of the Line of Lines (ſuppoſing the whole Line to be 10000) and it will give the true Diſtance between the Center, and the points of 25, in the Lines of Superficies.

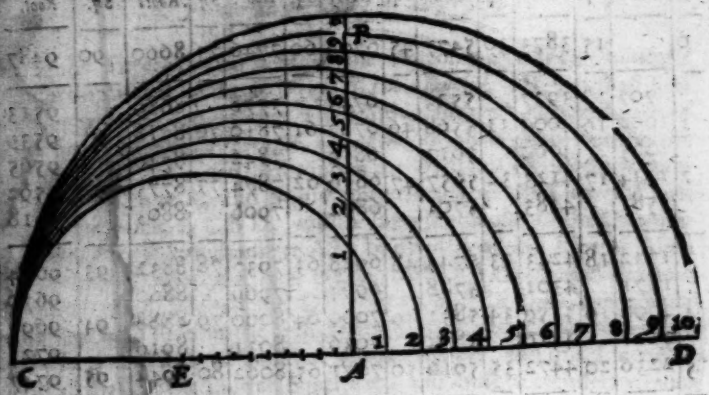
So, for the Diviſion of 30, put to 30 ſix Ciphers, and make it 30000000, whoſe Square Root is 5477. This (taken out of the Line of Lines) ſhall give the place for the Points of 30, in the Lines of Superficies. And the like reaſon holdeth for all the reſt, according to this following Table.

If any pleaſe to make uſe of a Diagonal Scale, equal to the Line of Lines, he may put eight Ciphers to the Number propoſed, and make the Table of Roots to five Places: So, his work will be more exact.

A Table of Square Roots *for the Diviſion of the Lines of* Superficies.

A Table of Square Roots for Division of the Line of Superficies.

Sq	Root.	Sq.	Root.	Sq	Root	Sq	Root.	Sq.	Root.	Sq	Root.	Sq.	Root.
0		15	3873	30	5477	45	6708	60	7746	75	8660	90	9487
	707		3937		5523		6745		7778		8689		9513
1	1000	16	4000	31	5568	46	6782	61	7810	76	8718	91	9539
	1225		4062		5612		6819		7842		8746		9565
2	1414	17	4123	32	5657	47	6856	62	7874	77	8775	92	9592
	1581		4183		5791		6892		7906		8803		9618
3	1732	18	4243	33	5744	48	6928	63	7937	78	8832	93	9644
	1871		4301		5788		6964		7969		8860		9670
4	2000	19	4359	34	5831	49	7000	64	8000	79	8888	94	9695
	2121		4416		5874		7036		8031		8916		9721
5	2236	20	4472	35	5916	50	7071	65	8062	80	8944	95	9747
	2345		4528		5958		7106		8093		8972		9772
6	2449	21	4582	36	6000	51	7141	66	8124	81	9000	96	9798
	2550		4637		6042		7176		8155		9028		9823
7	2646	22	4690	37	6083	52	7211	67	8185	82	9055	97	9849
	2739		4743		6124		7246		8216		9083		9874
8	2828	23	4796	38	6164	53	7280	68	8246	83	9110	98	9899
	2915		4848		6205		7314		8276		9138		9925
9	3000	24	4899	39	6245	54	7348	69	8307	84	9165	99	9950
	3082		4950		6285		7382		8337		9192		9975
10	3162	25	5000	40	6325	55	7416	70	8367	85	9219	100	10000
	3240		5050		6364		7450		8396		9247		
11	3317	26	5099	41	6403	56	7483	71	8426	86	9274		
	3391		5148		6442		7517		8556		9300		
12	3464	27	5196	42	6481	57	7550	72	8485	87	9327		
	3536		5244		6519		7583		8515		9354		
13	3606	28	5291	43	6557	58	7616	73	8544	88	9381		
	3674		5338		6595		7648		8573		9407		
14	3742	29	5385	44	6633	59	7681	74	8602	89	9434		
	3808		5431		6671		7714		8631		9460		
15	3873	30	5477	45	6708	60	7746	75	8660	90	9487		



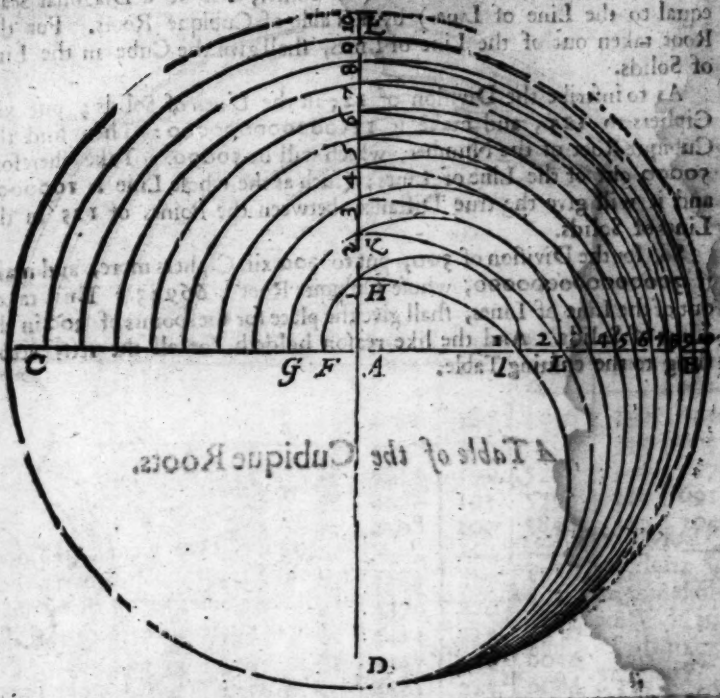
4. *To divide the Lines of Solids.*

SEeing like Solids do hold in the Proportion of their homologal Sides triplicated, if you ſhall find two mean Proportionals between the whole Side and each thouſandth part of the like Side: all of them cutting the ſame two right Lines, the former of thoſe Lines ſo cut, ſhall contain the Diviſions required.

Wherefore upon the Center A, and Semidiameter equal to the Line of Lines, deſcribe a Circle and divide it into 4 equal parts C E B D, drawing the croſs-Diameters C B E D. Then divide the Semidiameter A C, firſt into 10 equal parts, and between the whole Line A D and A F, the tenth part of A C, ſeek out two mean Proportional Lines A I and A H: again between A C and A G (being two Tenths of A B) ſeek out two mean Proportionals A L and A K, and ſo forward in the reſt. So ſhall the Line A B, be divided into 10 unequal parts.

Secondly, divide each tenth part of the Line A C into 10 more, and between the whole Line A D, and each of them, ſeek out two mean Proportionals as before: So ſhall the Line A B be divided now into an hundred unequal parts.

Thirdly,



Thirdly, if the Length will bear it, ſubdivide the Line A C once again, each part in ten more, and between the whole Line A D, and each Subdiviſion, ſeek two mean Proportionals as before. So ſhall the Line A B be now divided into 1000 parts. But the Ruler being ſhort, it ſhall ſuffice, if thoſe 10 which are neareſt the Center be expreſſed, the reſt be underſtood to be ſo divided, though actually they be divided into no more than 5 or 2, and this Line A B ſo divided ſhall be the Line of Solids, and muſt be transferred into the Sector: But let the Numbers ſet to them be only 1. 1. 1. 2. 3. &c. unto 10, as in the Example.

Or these Lines of Solids may otherwise be transferred into the Sector, out of the Line of Lines (or rather, out of a Diagonal Scale equal to the Line of Lines) by a Table of Cubique Roots. For the Root taken out of the Line of Lines, shall give the Cube in the Line of Solids.

As to inscribe the Division of 125 in the Lines of Solids; put xii. Ciphers to 125, and make it 125000000000000: Then find the Cubique Root of the Number, which will be 50000. Take therefore 50000 out of the Line of Lines; (such as the whole Line is 100000) and it will give the true Distance between the Points of 125 in the Lines of Solids.

So, for the Division of 300, put to 300 xii. Ciphers more, and make it 300000000000000, whose Cubique Root is 66943. This, taken out of the Line of Lines, shall give the place for the points of 300 in the Lines of Solids. And the like reason holdeth for all the rest, according to the ensuing Table.

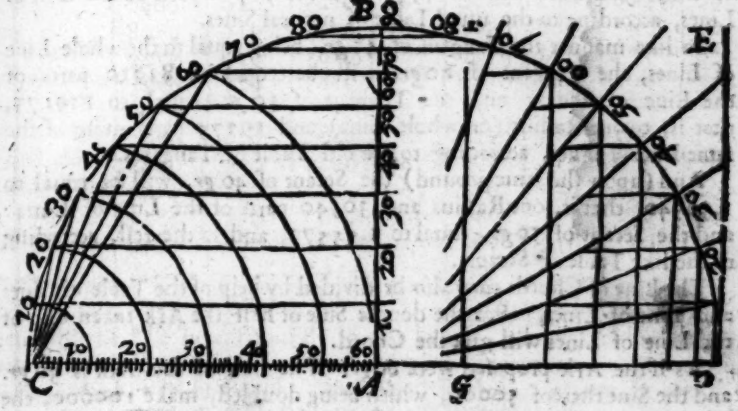
A Table of the Cubique Roots.

Cub	Root.	Cub	Root.	Cub.	Root.	Cub.	Root.	Cub.	Root.
0	0	20	2714	50	3684	125	5000	275	6502
	794	21	2758	52	3732	130	5065	280	6542
1	1000	22	2802	54	3779	135	5129	285	6580
	1144	23	2843	56	3825	140	5192	290	6619
2	1259	24	2884	58	3870	145	5253	295	6656
	1357	25	2924	60	3914	150	5313	300	6694
3	1442	26	2962	62	3957	155	5371	305	6751
	1518	27	3000	64	4000	160	5428	310	6767
4	1587	28	3036	66	4041	165	5484	315	6804
	1650	29	3072	68	4081	170	5539	320	6839
5	1709	30	3107	70	4121	175	5593	325	6875
	1765	31	3141	72	4160	180	5646	330	6910
6	1817	32	3174	74	4198	185	5698	335	6945
	1866	33	3207	76	4235	190	5748	340	6979
7	1912	34	3239	78	4272	195	5798	345	6913
	1957	35	3271	80	4308	200	5848	350	7047
8	2000	36	3301	82	4344	205	5896	355	7080
	2040	37	3332	84	4379	210	5943	360	7119
9	2080	38	3361	86	4414	215	5990	365	7146
	2117	39	3391	88	4447	220	6036	370	7179
10	2154	40	3419	90	4481	225	6089	375	7211
11	2223	41	3448	92	4515	230	6126	380	7243
12	2289	42	3476	94	4546	235	6171	385	7274
13	2351	43	3503	96	4578	240	6214	390	7306
14	2410	44	3530	98	4610	245	6257	395	7337
15	2466	45	3556	100	4641	250	6299	400	7368
16	2519	46	3583	105	4717	255	6341	405	7398
17	2571	47	3608	110	4791	260	6382	410	7428
18	2620	48	3634	115	4862	265	6423	415	7459
19	2668	49	3659	120	4931	270	6463	420	7488
20	2714	50	3684	125	5000	275	6502	425	7518

Cub.	Root.	Cub.	Root.	Cub.	Root.	Cub.	Root.
425	7518	575	8315	725	8983	875	9564
430	7547	580	8539	730	9004	880	9580
435	7576	585	8363	735	9024	885	9600
440	7605	590	8387	740	9045	890	9619
445	7634	595	8410	745	9065	895	9636
450	7663	600	8434	750	9085	900	9654
455	7691	605	8457	755	9105	905	9672
460	7719	610	8480	760	9125	910	9690
465	7747	615	8504	765	9145	915	9708
470	7774	620	8527	770	9165	920	9725
475	7802	625	8549	775	9185	925	9743
480	7829	630	8572	780	9205	930	9761
485	7856	635	8595	785	9224	935	9778
490	7883	640	8617	790	9244	940	9795
495	7910	645	8640	795	9263	945	9813
500	7937	650	8662	800	9283	950	9830
505	7963	655	8684	805	9302	955	9847
510	7989	660	8706	810	9321	960	9864
515	8015	665	8728	815	9340	965	9881
520	8041	670	8750	820	9359	970	9898
525	8067	675	8772	825	9378	975	9915
530	8092	680	8793	830	9397	980	9932
535	8118	685	8815	835	9416	985	9949
540	8143	690	8836	840	9435	990	9966
545	8168	695	8857	845	9454	995	9983
550	8193	700	8879	850	9472	1000	10000
555	8217	705	8900	855	9491		
560	8242	710	8921	860	9509		
565	8267	715	8942	865	9529		
570	8291	720	8962	870	9546		
575	8315	725	8983	875	9564		

5. *To divide the Lines of Sines and Tangents on the Side of the Sector.*

UPon the Center A, and Semidiameter equal to the Line of Lines, describe a Semicircle A B C D, with A B, perpendicular to the Diameter C D. Then divide the Quadrant C B, B D, each of them into 90, and subdivide each degree into two parts: For so if streight Lines be drawn parallel to the Diameter C D, through these 90, and their Subdivisions, they shall divide the Perpendicular A B unequally into 90.



And this A B (so divided) shall be the Line of Sines, and must be transferred into the Sector. The Number set to them are to be 10, 20, 30, *&c.* unto 90, as in the Example.

If now in the point D, unto the Diameter C D, we shall raise a Perpendicular D E, and to it draw streight Lines from the Center A, through each Degree of the Quadrant D B, these streight Lines shall be Secants, and this Perpendicular so divided by them shall be the Line of Tangents, and must be transferred unto the Side of the Sector. The Number set to them, are so be 10, 20, 30, *&c.* as in the Example.

If between A and D, another streight Line G F be drawn parallel to D E, it will be divided by those Lines from the Center in like sort as

D E is divided, and it may ſerve for a leſſer Line of Tangents, to be ſet on the Edge of the Sector.

If the Compaſſes ſhall be extended from C to each degree of the Quadrant C B, and thoſe Extents transferred into one Line (C A) this Line C A ſo divided into 60 (or rather into 90 *gr.*) ſhall be a Line of Chords, and may be ſet on ſome void place of the Sector.

Theſe Lines of Sines and Tangents, may yet otherwiſe be transferred into the Sector out of the Line of Lines (or rather out of a Diagonal Scale equal to the Line of Lines,) by Tables of Natural Sines and Tangents.

For the Sine of 90 *gr.* being equal to the whole Line of Lines of 100000 parts, the Sine of 30 *gr.* will be equal to 50000 (half the Line of Lines;) and the Sine of 45 *gr.* equal to 70710 parts of the Line of Lines, according to the uſual Table of natural Sines.

In like manner the Tangent of 45 *gr.* being equal to the whole Line of Lines, the Tangent of 40 *gr.* will be equal to 83910 parts of the Line of Lines: and the Tangent of 50 *gr.* equal to 119175, that is, to one Radius (or whole Line) and 19175 parts more of the ſame Line of Lines, according to the old Table of Tangents.

And (upon the ſame ground) the Secant of 40 *gr.* will be equal to 1.30540, that is, one Radius and 30540 parts of the Line of Lines: and the Secant of 50 *gr.* equal to 1.55572, and ſo the reſt, according to the like Table of Secants.

The Line of Chords may alſo be divided by help of the Table of Sines and Line of Lines. For the double Sine of half the Ark taken out of the Line of Lines will give the Chord.

As if the Ark propoſed were 60 *gr.* The half of this Ark is 30 *gr.* and the Sine thereof 50000, which being doubled, make 100000, the whole Line of Lines, equal to a Chord of 60 *gr.*

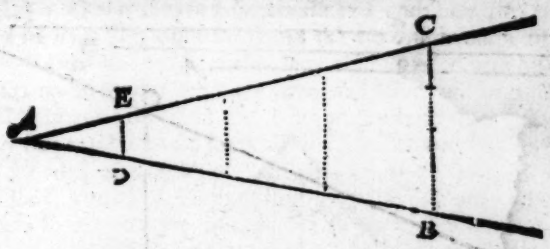
So for the Chord of 90 *gr.* the half Ark is 45 degrees, and the Sine thereof 70710, which being doubled, make 1414240, that is, one Radius, and 41420 parts of the Line of Lines, equal to the Chord of 90 *gr.* required.

6. *To ſhew the Ground of the Sector.*

LEt A B, A C, repreſent the Legs of the Sector; then ſeeing theſe two A B, A C are equal, and their Sections A D, A E, alſo equal, they ſhall be cut proportionally: and if we draw the Lines B C, D E, they will be parallel by *Prop.* 2. *Lib.* 6. of *Euclid*, and ſo the Triangles

A B C

A B C, A D E, ſhall be equiangled, by reaſon of the common Angle at A, and the equal Angles at the Baſe, and therefore ſhall have the Sides proportional about thoſe equal Angles, by *Prop.* 4. *Lib.* 6. of *Euclid.*



The Side A D ſhall be to the Side A B, as the Baſis, D E, unto the parallel Baſis B C, and by converſion A B ſhall be unto A D, as B C unto D E; and by permutation A D ſhall be unto D E, as A B to B C, *&c.* So that if A D be the fourth part of the Side A B, then D E ſhall alſo be the fourth part of his parallel Baſis B C. The like reaſon holdeth in all other Sections.

7. *To ſhew the general Uſe of the Sector.*

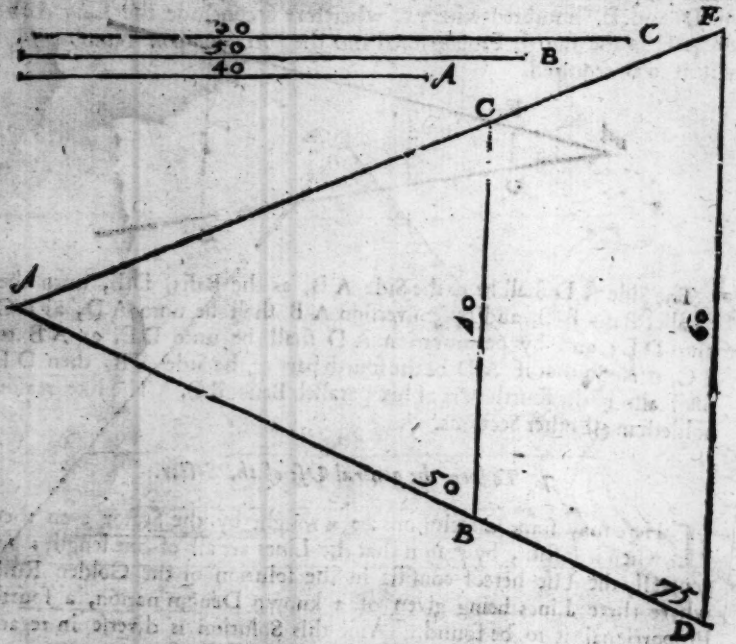
THere may ſome Concluſions be wrought by the Sector even then when it is ſhut, by reaſon that the Lines are all of one length: but generally the Uſe hereof conſiſts in the ſolution of the Golden Rule, where three Lines being given of a known Denomination, a fourth Proportional is to be found. And this Solution is diverſe in regard both of the Lines and of the Entrance into the Work.

The Solution in regard of the Lines is ſometimes ſimple, as when the Work is begun and ended upon the ſame Lines. Sometimes it is compound, as when it is begun on one kind of Lines and ended on another. It may be begun upon the Lines of Lines, and finiſhed upon the Lines of Superficies. It may begin on the Sines, and end on the Tangents.

The Solution in regard of the Entrance into the work, may be either with a Parallel, or elſe Lateral on the Side of the Sector, I call it Parallel Entrance, or entring with a Parallel, when the two Lines of the firſt Denomination are applied in the Parallels, and the third Line, and that which is ſought for, are on the ſide of the Sector: I call it lateral Entrance, or entring on the ſide of the Sector, when the two Lines of

the

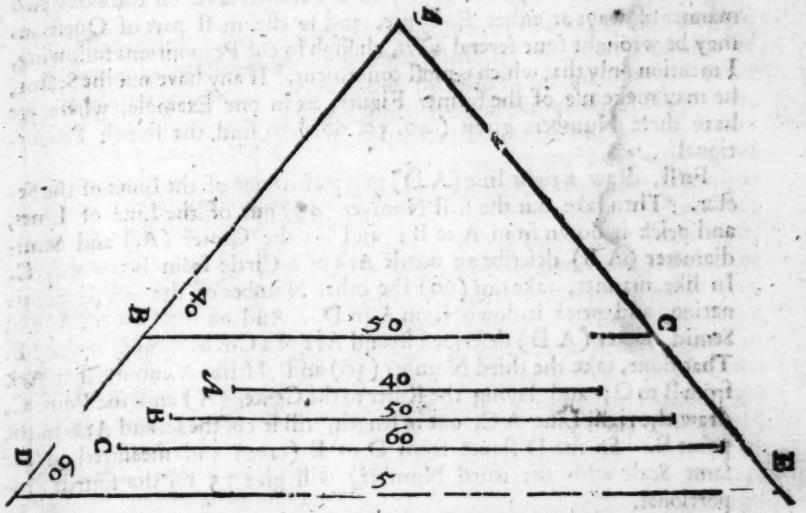
the first Denomination are on the side of the Sector, and the third Line, and that which is to be found out do stand in the Parallels.



As for Example, let there be given three Lines A, B, C, to which I am to find a fourth Proportional, let A measured in the Line of Lines, be 40, B 50, and C 60, and suppose the Question be this: If 40 Months give 50 pounds, what shall 60? Here are Lines of two Denominations, one of Months, another of Pounds, and the first, with which I am to enter, must be that of 40 Months. If then I would enter with a Parallel, first I take A, the Line of 40, and put it over as a Parallel in 50, reckoned in the Line of Lines, on either side of the Sector from the Center, so as it may be the Base of an Isosceles Triangle B A C, whose Sides A B, A C are equal to B, the Line of the second Denomination.

Then

Then the Sector being thus opened, I take C the Line of 60, between the Feet of the Compaſſes, and carrying them parallel to B C, I find them to croſs the Lines A B, A C, on the ſide of the Sector in D and E, numbred with 75, wherefore I conclude the Line A D or A E is the fourth Proportional and the correſpondent Number 75, which was required.



But if I would enter on the Side of the Sector, then would I diſpoſe the Lines of the firſt Denomination A and C in the Line of Lines, on both ſides of the Sector in A B, A C, and in A D, A E, ſo as they ſhould all meet in the Center A, and then taking B the Line of the ſecond Denomination, put it over as a Parallel in B C, that it may be the Baſis of the Iſoſcheles Triangle B A C (whoſe Sides A B, A C, are equal to A the firſt Line of the firſt Denomination) for ſo the Sector being thus opened, the other Parallel from D to E, ſhall be the fourth Proportional which was required, and if it be meaſured with the other Lines, it ſhall be 75, as before.

In both theſe manners of Operations, the two firſt Lines do ſerve to open the Sector to his due Angle, the Difference between them is eſpecially this,

this, that in Parallel Entrance, the two Lines of the first Denomination, are placed in the Parallels B C, D E, and in Lateral Entrance they are placed on both Sides of the Sector, in A B, A D, and in A C, A E.

Now in simple solution which is begun and ended upon the same kind of Lines, it is all one which of the two latter Lines be put in the second or third place. As in our Example we may say, *As* 40 *are to* 50, *so* 60 *unto* 75, or else, *As* 40 *are to* 60, *so* 50 *unto* 75. And hence it cometh, that we may enter both with a Parallel, and on the Sides two manner of ways at either Entrance, and so the most part of Questions may be wrought four several ways, though in the Propositions following, I mention only that which is most convenient. If any have not the Sector, he may make use of the former Figure, as in our Example, where we have three Numbers given (40. 50. 60.) to find the fourth Proportional.

First, draw a right line (A D) to represent one of the Lines of the Sector. Then take out the first Number (48) out of the Line of Lines, and prick it down from A to B; and on the Center (A,) and Semidiameter (A B) describe an occult Ark of a Circle from B towards C. In like manner, take out (60) the other Number of the first Denomination, and prick it down from A to D. And on the Center (A) and Semidiameter (A D) describe a second Ark of a Circle, from D toward E. That done, take the third Number (50) and inscribe it into the first Ark from B to C; and laying the Ruler to the Center (A) and the Point C, draw the right Line A C, out in length, till it cut the second Ark in the point E. So the Distance from D to E (taken and measured in the same Scale with the third Number) will give 75 for the fourth Proportional.

Thus much for the general Use of the Sector, which being considered, and well understood, there is nothing hard in that which followeth.

CHAP.

CHAP. II.

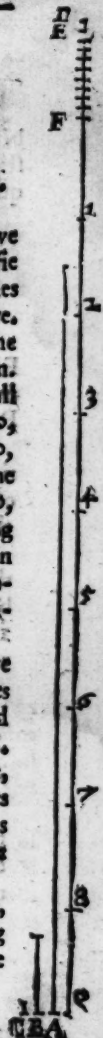
The Use of the Scale of Lines.

1. *To set down a Line, resembling any given Parts or Fraction of Parts.*

THe Lines of Lines are divided actually into 100 parts, but we have put only 10 Numbers in them. These we would have to signifie either themselves alone, or ten times themselves, or an hundred times themselves, or a thousand times themselves, as the matter shall require. As if the Numbers given be no more than 10, then we may think the Lines only divided into ten parts according to the number set to them. If they be more than 10, and not more than 100, then either Line shall contain 100 parts, and the Numbers set by them shall be in value 10, 20, 30, *&c.* as they are divided actually. If yet they be more than 100, then every part must be thought to be divided into 10, and either Line shall be 1000 parts, and the Numbers set to them shall be in value 100, 200, 300, and so forward still increasing themselves by 10. This being presuppoſed, we may number the Parts and Fraction of Parts given in the Line of Lines; and taking out the Distance with a Pair of Compasses, set it by, for the Line so taken shall resemble the Number given.

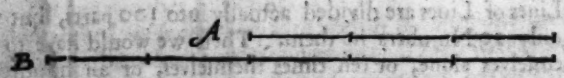
In this manner may we set down a Line resembling 75, if either we take 75 out of the hundred parts, into which one of the Line of Lines is actually divided, and note it in A, or 7½ of the first 10 parts, and note it in B, or only ¾ of one of those hundred Parts, and note it in C. Or if this be either too great or too small, we may run a Scale at pleasure, by opening the Compass to some small distance, and running it ten times over, then opening the Compass to these ten, run them over nine times more, and set Figures to them as in this Example, and out of this we may take what parts we will as before.

To this end I have divided the Line of Inches on the Edge of the Sector, so as one Inch conteineth 8 parts, another 9, another 10, *&c.* according as they are figured, and as they are distant from the other end of the Sector, that so we might have the better Estimate,



2. *To increase a Line in a given Proportion.*
3. *To diminish a Line in a given Proportion.*

TAke the Line given with a pair of Compasses, and open the Sector, so as the Feet of the Compasses may stand in the point of the Number given, then keeping the Sector at this Angle, the Parallel Distance of the points of the Number required, shall give the Line required.



Let A be a Line given to be increased in the Proportion of 3 to 5. First, I take the Line A with the Compasses, and open the Sector till I may put it over in the Points of 3 and 3, so the Parallel between the Points of 5 and 5, doth give me the Line B, which was required.

In like manner, if B be a Line given to be diminished in the Proportion of 5 to 3, I take the Line B: and to it open the Sector in the points of 5 and 5, so the Parallel between the Points of 3 and 3, doth give me the Line A, which was required.

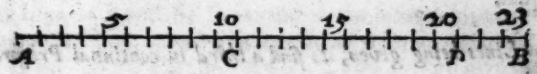
If this manner of work doth not suffice, we may multiply or divide the Numbers given by 2, or 3, or 4, *&c.* And so work by their Numbers *æqui-multiplices*, as for 3 and 5, we may open the Sector in 6 and 10, or else in 9 and 15, or else in 12 and 20, or in 15 and 25, or in 18 and 30, *&c.*

4. *To divide a Line into any number of Parts given.*

TAke the Line given, and open the Sector according to the length of the said Line in the points of the parts, whereinto the Line should be divided, then keeping the Sector at this Angle, the Parallel Distance between the points of 1 and 1 shall divide the Line given into the Parts required.

Let

Let A B be the Line given, to be divided into five parts, first I take this Line A B, and to it open the Sector in the point of 5 and 5, the Parallel between the points of 1 and 1, doth give me the Line A C, which doth divide it into the parts required.



Or let the like Line A B be to be divided into twenty three parts. First, I take out the Line and put it upon the Sector in the points of 23, then may I by the former Proposition diminish it in A C, C D, in the Proportion of 23 to 10, and after that divide the Line A C into 10, &c. as before.

5. *To find a Proportion between two or more right Lines given.*

TAke the greater Line given, and according to it open the Sector in the points of 100 and 100, then take the lesser Lines severally, and carry them parallel to the greater, till they stay in like points, so the Number of points wherein they stay, shall shew their Proportion unto 100.

Let the Lines given be A B, C D, first I take the Line C D, and to it open the Sector in the points of 100 and 100, then keeping the Sector at this Angle, I enter the lesser Line A B, parallel to the former, and find it to cross the Lines of Lines in the points of 60. Wherefore the Proportion of A B to C D, is as 60 to 100.

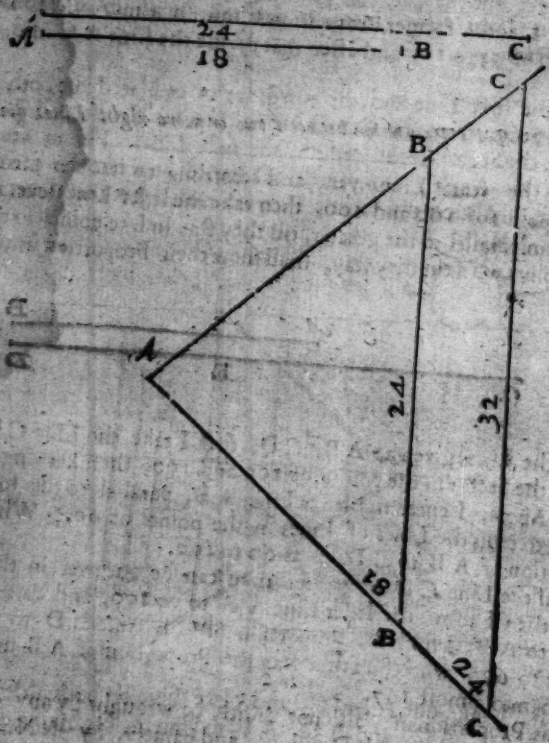
Or if the Line C D be greater than can be put over in the Points of 100, then I admit the lesser Line A B to be 100, and cutting off C E equal to A B, I find the Proportion of C E unto E D to be as 100, almost to 67; wherefore this way the Proportion of A B unto C D, is as 100 unto almost 167.

This Proportion may also not unfitly be wrought by any other Number, that admits several Divisions, and namely, by the Numbers of 60.

And so the lesser Line will be found to be 36, which is as before in lesser Numbers, as 3 unto 5. It may also be wrought without opening the Sector. For if the Lines between which we seek a proportion, be applied to the Lines of Lines (or any other Scale of equal parts) there will be such Proportion found between them, as between the Lines to which they are equal.

6. *Two Lines being given, to find a third in continual Proportion.*

First place both the Lines given, on both sides of the Sector from the Center, and mark the terms of their Extension, then take out the



second

ſecond Line again, and to it open the Sector, in the terms of the firſt Line, ſo keeping the Sector at this Angle, the parallel Diſtance between the terms of the ſecond Line, ſhall be the third Proportional.

Let the two Lines given be A B, A C, which I take out and place on both ſides of the Sector, ſo as they all meet in the Center A, let the terms of the firſt Line be B and B, the terms of the ſecond C and C. Then do I take out A G the ſecond Line again, and to it open the Sector in the terms B B. So the Parallel between C and C doth give me the third Line in continual Proportion. For as A B is unto A C, ſo B B equal to A C, is unto C C.

7. *Three Lines being given, to find the fourth in diſcontinual Proportion.*

HEre the firſt Line and the third are to be placed on both ſides of the Sector from the Center, then take out the ſecond Line, and to it open the Sector in the terms of the firſt Line. For ſo keeping the Sector at this Angle, the parallel Diſtance between the terms of the third Line, ſhall be the fourth Proportional. Let the three Lines given be A, B, C.

Firſt, I take out A and C, and place them on both ſides of the Sector, in A B, A C, and A D, A E, laying the beginning of both Lines

at the Center A, then do I take out B the ſecond Line, according to it I open the Sector in B and C, the terms of the firſt Line: ſo the Parallel between D and E, doth give me the fourth Proportional which was required.

As in *Arithmetick*, it ſufficeth if the firſt and third Number given be of one Denomination, the ſecond and the fourth which is required be of another. For one and the ſame Denomination is not required neceſſarily in them all. So in *Geometry*, it ſufficeth if the Sides A B, A D, reſembling the firſt and third Lines given be meaſured in one Scale, and the Parallels B C, D E be meaſured in another. Wherefore knowing the Proportion of A the firſt Line, and C the third Line by the fifth Propoſition before. Which is here as 8 to 12, and deſcending in leſſer Numbers, is as 4 to 6, or as 2 to 3, or aſcending into greater Numbers, as 16 unto 24, or 18 to 27, or 20 to 30, or 30 to 45, or 40 to 60, *&c.* If the Sector be opened in the points of 8 and 8, to the quantity of B, the ſecond Line given, then a Parallel between 12 and 12, ſhall give D E, the fourth Line required. So likewiſe if it be opened in 4 and 4, then a Parallel between 6 and 6; or if in 16 and 16, then a Parallel between 24 and 24 ſhall give the ſame D E: and ſo in the reſt.

8. *To divide a Line in ſuch ſort as another Line is before divided.*

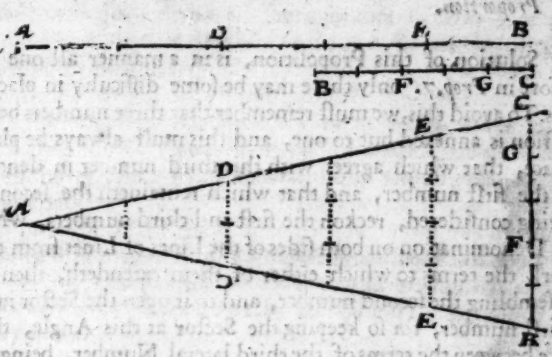
FIrſt, take out the Line given, which is already divided, and laying it on both ſides of the Sector from the Center; mark how far it extendeth. Then take out the ſecond Line which is to be divided, and to it open the Sector in the terms of the firſt Line. This done, take out the parts of the firſt Line, and place them alſo on the ſame ſide of the Sector from the Center. For the Parallels taken in the terms of theſe parts ſhall be the correſpondent parts in the Line which is to be divided.

Let A B, be a Line divided in D and E, and B C the Line which I am to divide in ſuch ſort, as A B is divided.

First, I take Line A B, and place it on the Line of Lines in A B, A C, both from the Center A, then take I out the ſecond B C, and to it open the Sector in B and C, the terms of the firſt Line. The Sector thus opened to his due Angle, I take out A D and A E, the parts of the firſt Line A B, and place them alſo on both the ſides of the Sector A D, A E, ſo the Parallel D D giveth me B F, and the Parallel E E

giveth

giveth B G, and the Line B C is divided in F and G, as is the other Line A B in D and E, which was required.



If the Line A B were longer than one of the Sides of the Ruler, then ſhould I find what proportion it hath to his parts A D, A E, and that known, I may work as before in the former Propoſition.

9. *Two Numbers being given, to find a third in continual Proportion.*

FIrſt reckon the two numbers given on both ſides of the Lines of Lines from the Center, and mark the terms to which either of them extendeth, then take out a Line reſembling the ſecond number again, and to it open the Sector in the terms of the firſt number, for ſo keeping the Sector at this Angle, the Parallel Diſtance between the terms of the ſecond lateral Number, being meaſured in the ſame Scale, from whence his Parallel was taken, ſhall give the third Number Proportional.

Let the two Numbers given be 18, 24. Theſe being reſembled in Lines, the work will be in a manner all one with that in *Prop.* 6. and ſo the third Proportional number will be found to be 32.

10. *Three*

10. *Three Numbers being given, to find a fourth in diſcontinual Proportion.*

THe Solution of this Propoſition, is in a manner all one with that before in *Prop.*7. only there may be ſome difficulty in placing of the numbers. To avoid this, we muſt remember that three numbers being given, the queſtion is annexed but to one, and this muſt always be placed in the third place, that which agrees with this third number in denomination, ſhall be the firſt number, and that which remaineth the ſecond number. This being conſidered, reckon the firſt and third numbers, which are of the firſt Denomination on both ſides of the Lines of Lines from the Center, and mark the terms to which either of them extendeth, then take out a Line reſembling the ſecond number, and to it open the Sector in the terms of the firſt number, for ſo keeping the Sector at this Angle, the parallel Diſtance between the terms of the third lateral Number, being meaſured in the ſame Scale from whence his Parallel was taken, ſhall give the fourth number Proportional.

As if a queſtion were propoſed in this manner, 10 yards coſt 8 *l.* how many yards may we buy for 12 *l.* here the queſtion is annexed to 12; and therefore it ſhall be the third number, and becauſe 8 is of the ſame denomination, it ſhall be the firſt number, then 10 remaining, it muſt be the ſecond number, ſo will they ſtand in this order, 8, 10, 12. Theſe being reſembled in Lines, the work will be in a manner the ſame with that in *Prop.*7. and the fourth Proportional number will be found to be 15: for *as* 8 *are to* 10, *ſo* 12 *unto* 15.

And this holdeth in direct Proportion; where as the firſt number is to the ſecond, ſo the third to the fourth. So that if the third number be greater than the firſt, the fourth will be greater than the ſecond; or if the third number be leſs than the firſt, the fourth will be leſs than the ſecond, but in reciprocal Proportion, commonly called the *Back Rule*, where, by how much the firſt number is greater than the third, ſo much the ſecond will be leſs than the fourth, or by how much the firſt number is leſs than the third, ſo much the ſecond will be greater than the fourth; the manner of working muſt be contrary, that is, the Sector is to be opened in the terms of the third number: and the Parallel reſembling the number required, is to be found between the terms of the firſt number, the reſt may be obſerved as before, as for example.

If twelve men would raise a Frame in ten days, in how many days would eight men raise the same Frame? Here, because the fewer men would require longer time, though the numbers be 12, 10, 8, yet the fourth Proportional will be found to be 15.

So if 60 Yards of three quarters of a Yard in bredth would hang round about a room, and it were required to know how many Yards of half a Yard in bredth would serve for the same room. The fourth Proportional would be found to be 90.

So if to make a Foot superficial 12 inches in bredth do require 12 inches in length, and the bredth being 16 inches, it were required to know the length. Here, because the more bredth, the less length, the fourth Proportional will be found to be 9.

So if to make a solid Foot, a Base of 144 inches, require 12 inches in height, and a Base given being 216 inches, it were required to know how many inches it shall have in height. The fourth Proportional would be found to be 8.

This last Proposition of finding a fourth Proportional Number may be wrought also by the Lines of Superficies, and by the Lines of Solids.

CHAP. III.

The Use of the Lines of Superficies.

1. *To find a Proportion between two or more like Superficies.*

TAke one of the sides of the greater Superficies given, and according to it open the Sector in the points of 100 and 100 in the Lines of Superficies, then take the like sides of the lesser Superficies severally, and carry them parallel to the former, till they stay in like points, so the number of points wherein they stay, shall shew their Proportion unto 100.

Let A and B, be the sides of like Superficies, as the sides of two Squares, or the Diameters of two Circles, first I take the side A, and to it open the Sector in the points of 100, then keeping the Sector to this Angle, I enter the lesser side B, parallel to the former, and find it to cross the Lines of Superficies in the points of 40, wherefore the Proportion of the Superficies, whose side is A, to that whose side is B, is as 100 unto 40, which is in lesser number as 5 unto 2.

This Proposition might have been wrought by 60, or any other Number that admits several Divisions. It may also be wrought without opening the Sector, for if the sides of the Superficies given be applied to the Lines of Superficies, beginning always at the Center of the Sector, there will be such Proportion found between them, as between the number of parts whereon they fall.

2. *To augment a Superficies in a given Proportion.*
3. *To diminish a Superficies in a given Proportion.*

TAke the side of the Superficies, and to it open the Sector in the points of the numbers given; then keeping the Sector at that Angle, the parallel distance between the points of the number required, shall give the like side of the Superficies required.

Let

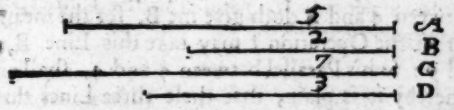
Let A be the ſide of a Square, to be augmented in the Proportion of 2 to 5. Firſt, I take the ſide A, and put it over in the Lines of Superficies in 2 and 2; ſo the Parallel between 5 and 5, doth give me the ſide B, on which if I ſhould make a Square, it would have ſuch Proportion to the Square of A, as 5 unto 2.

In like manner, if B were the Semidiameter of a Circle to be diminiſhed in the Proportion of 5 unto 2, I would take out B, and put it over in the Lines of Superficies in 5 and 5; ſo the Parallel between 2 and 2 would give me A; on which Semidiameter if I ſhould make a Circle, it would be leſs than the Circle made upon the Semidiameter B, in ſuch Proportion as 2 is leſs than 5.

For variety of work, the like caution may be here obſerved to that which we gave in the third Propoſition of Lines.

4. *To add one like Superficies to another.*
5. *To ſubtract one like Superficies from another.*

FIrſt, the Proportion between like ſides of the Superficies given, is to be found by the firſt Propoſition of Superficies, then add or ſubtract the numbers of thoſe Proportions, and accordingly augment or diminiſh by the former Propoſition.



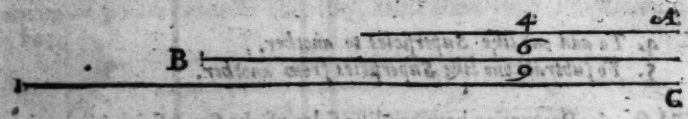
As if A and B were the ſide of two Squares, and it were required to make a third Square equal to them both. Firſt the Proportion between the Squares of A and B, would be found to be as 100 unto 40, or in the leſſer numbers as 5 to 2; then becauſe 5 and 2 added do make 7, I augment the ſide A in the Proportion of 5 to 7, and it will produce the ſide C, on which if I make a Square, it will be equal to both the Squares of A and B, which was required.

In like manner A and B being the ſides of two Squares, if it were required to ſubtract the Square of B, out of the Square of A, and to make a Square equal to the Remainder, here the Proportion being as 5 to 2, becauſe 2 taken out of 5, the Remainder is 3, I would diminiſh the ſide A in the Proportion of 5 to 3, and ſo I ſhould produce the ſide

D, on which if I make a Square, it will be equal to the Remainder, when the Square of B is taken out of the Square of A, that is, the two Squares made upon B and D, shall be equal to the first Square made upon the side A.

6. To find a mean Proportional between two Lines given.

FIrst find what Proportion is between the Lines given, as they are Lines, by the fifth Proposition of Lines, then open the Sector in the Lines of Superficies, according to his Number, to the quantity of the one, and a Parallel taken between the points of the Number belonging to the other Line shall be the mean Proportional.



Let the Lines given be A and C. The Proportion between them (as they are Lines) will be found, by the fifth Proposition of Lines, to be, as 4 to 9. Wherefore, I take the Line C, and put it over to the Lines of Superficies between 9 and 9, and keeping the Sector at this Angle, his Parallel between 4 and 4 doth give me B, for the mean Proportional. Then for proof of the Operation I may take this Line B, and put over between 9 and 9: so his Parallel between 4 and 4, shall give me the first Line A. Whereby it is plain, that these three Lines do hold in continual Proportion; and therefore B is a mean Proportional between A and C, the extremes given.

Upon the finding out of this mean Proportion, depend many Corollaries, as

To make a Square equal to a Superficies given.

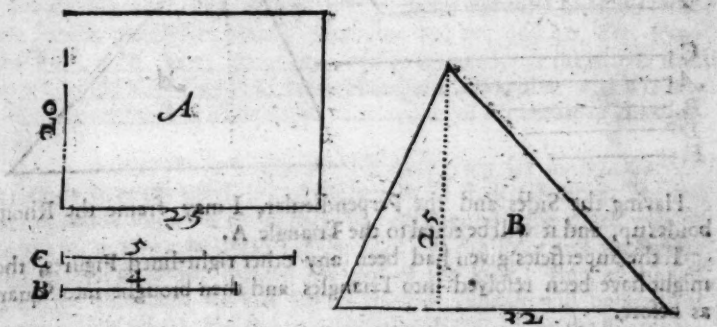
IF the Superficies given be a rectangle Parallelogram, a mean Proportional between the two unequal Sides shall be the Side of his equal Square.

If it shall be a Triangle, a mean Proportion between the Perpendicular and half the Base shall be the Side of his equal Square. If it shall be any other right-lined Figure, it may be resolved into Triangles, and so

a Side of a Square found equal to every Triangle, and these being reduced into one equal Square, it shall be equal to the whole right-lined Figure given.

To find a Proportion between Superficies, though they be unlike one to the other.

IF to every Superficies we find the side of his equal Square, the Proportion between these Squares shall be the Proportion between the Superficies given.



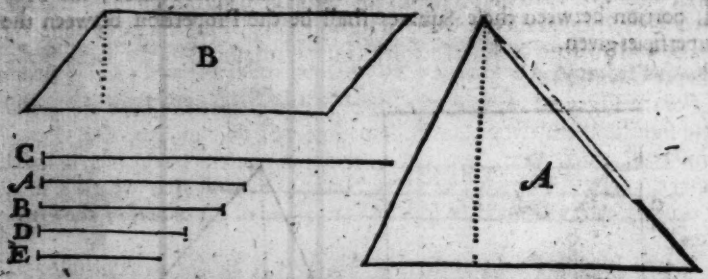
Let the Superficies given be the oblong A, and the Triangle B. First between the unequal Sides of A, I find a mean Proportional, and note it in C: This is the side of a Square equal unto A. Then between the Perpendicular of B, and half his Base, I find a mean Proportional, and note it in B: this is the side of a Square equal to B: but the Proportion between the Squares of C and B, will be found, by the first Proposition of Superficies to be as 5 to 4: and therefore this is the Proportion between those given Superficies.

To make a Superficies, like to one Superficies, and equal to another.

LEt the one Superficies given be the Triangle A, and the other the Rhomboides B; and let it be required to make another Rhomboides like to B, and equal to the Triangle A.

First,

Firſt, between the Perpendicular and the Baſe of B, I find a mean Proportional, and note it in B, as the ſide of his equal Square, then between the Perpendicular of the Triangle A, and half his Baſe, I find a mean Proportional, and note it in A, as the ſide of his equal Square. Wherefore now as the ſide B is to the ſide A, ſo ſhall the ſides of the Rhomboides given be to C and D, the ſides of the Rhomboides required, and his Perpendicular alſo to E, the Perpendicular required.



Having the Sides and the Perpendicular, I may frame the Rhomboides up, and it will be equal to the Triangle A.

If the Superficies given had been any other right-lined Figures, they might have been reſolved into Triangles and then brought into Squares as before.

Many ſuch Corollaries might have been annexed, but the means of finding a mean Proportional being known, they all follow of themſelves.

7. *To find a mean Proportional between two Numbers given.*

Firſt, reckon the two Numbers given on both ſides of the Lines of Superficies, from the Center, and mark the terms whereunto they extend; then take a Line out of the Line of Lines, or any other Scale of equal parts reſembling one of thoſe Numbers given, and put it over in the terms of his like Number in the Lines of Superficies; for ſo keeping the Sector at this Angle, the Parallel taken from the terms of the other Number and meaſured in the ſame Scale from which the other Parallel was taken, ſhall here ſhew the Mean Proportional which was required.

Let the Numbers given be 4 and 9. If I ſhall take the Line A in the

the Diagram of the ſixth Propoſition reſembling 4, in a Scale of equal parts, and to it open the Sector in the terms of 4 and 4, in the Lines of Superficies, his Parallel between 9 and 9 doth give me B for the Mean Proportional. And this meaſured in the Scale of equal parts doth extend to *6*, which is the Mean Proportional Number between 4 and 9: For as 4 to 6, ſo 6 to 9.

In like manner, if I take the Line C, reſembling 9, in a Scale of equal parts, and to it open the Sector in the terms of 9 and 9, in the Lines of Superficies, his Parallel between 4 and 4 doth give me the ſame Line B, which will prove to be 6, as before, if it be meaſured in the ſame Scale whence C was taken.

For the Figures 1, 2, 3, 4, *&c.* here ſet down upon the Line, do ſometime ſignifie themſelves alone: ſometime 10, 20, 30, 40, *&c.* ſometime 100, 200, 300, 400, *&c.* and ſo forward, as the matter ſhall require. The firſt Figure of every Number is alway that which is here ſet down: the reſt muſt be ſupplied according to the nature of the Queſtion.

If you ſuppoſe Pricks under the Number given (as in Arithmetical Extraction) and the laſt Prick to the left hand ſhall fall under the laſt Figure (which will be as oft as there be odd Figures), the unite will be beſt placed at 1, in the middle of the Line; ſo the Root and the Square will both fall forward, toward the end of the Line. But, if the laſt Prick ſhall fall under the laſt Figure but one (which will be as oft as there be even Figures) then the unite may be placed at 1 in the beginning of the Line, and the Square in the ſecond length: or the unite may be placed at 10, in the end of the Line, ſo the Root and the Square will both fall backward, toward the middle of the Line.

8. *To find the Square Root of a Number.*

9. *The Root being given, to find the Square Number of that Root.*

IN the Extraction of a Square Root it is uſual to ſet Pricks under the firſt Figure, the third, the fifth, the ſeventh, and ſo forward, beginning from the right hand toward the left, and as many Pricks as fall to be under the Square Number given, ſo many Figures ſhall be in the Root: ſo that if the Number given be leſs than 100, the root ſhall be only of one Figure; if leſs than 10000, it ſhall be but two Figures; if leſs than 1000000, it ſhall be three Figures, *&c.*

Thereupon the Lines of Superficies are divided firſt into an hundred parts,

parts, and if the Number given be greater than 100, the first Division (which before did signifie only one) must signifie 100, and the whole Line shall be 10000 parts: if yet the number given be greater than 10000, the first Division must now signifie 10000; and the whole Line be esteemed at 1000000 parts: and if this be too little to express the Number given, as oft as we have recourse to the beginning, the whole Line shall increase itself an hundred times.

By these means if the last Prick to the left hand shall fall under the last Figure, which will be as oft as there be odd Figures, the Number given shall fall out between the Center of the Sector and the tenth Division: but if the last Prick shall fall under the last Figure but one, which will be as oft as there be even Figures, then the Number given shall fall out between the tenth Division and the end of the Sector.

This being considered, when a Number is given, and the Square Root is required, take a pair of Compasses, and setting one Foot in the Center, extend the other to the term of the number given in one of the Lines of Superficies; for this Distance applied to one of the Lines of Lines, shall shew what the Square Root is, without opening the Sector.

Thus 36 doth give a Root of 6; and 360, a Root of almost 19: and 3600, a Root of 60; and 36000, a Root of 189, *&c.*

In like manner, the nearest Root of 725 is here found to be (about) 27, the nearest Root of 7250, about 85: the nearest of 72500, about 269; and the nearest Root of 725000, about 851: And so in the rest.

On the contrary, a Number given may be squared, if first we extend the Compasses to the Number given in the Lines of Lines, and then apply that Distance to the Lines of Superficies, as may appear by the former Examples.

10. *Three Numbers being given, to find the fourth in a duplicated Proportion.*

IT is plain (by *Euclid. Lib. 6. Prop.* 19 & 20.) that like Superficies doth hold in a duplicated Proportion of their homologal Sides, whereupon a question being moved concerning Superficies and their Sides: It is usual in Arithmetick, that the Proportion be first duplicated before the Question be resolved, which is not necessary in the Use of the Sector, only

only the Numbers which do signifie Superficies, must be reckoned in the Lines of Superficies, and they which signifie the Sides of Superficies, in the Lines of Lines, after this manner.

If a Question be made concerning a Superficies, the two Numbers of the first Denomination must be reckoned in the Lines of Lines: and the Sector opened in the terms of the first Number to the quantity of a Line out of the Scale of Superficies resembling the second Number; so his Parallels taken between the terms of the third Number, being measured in the same Scale of Superficies, shall give the Superficial Number which was required.

As if a Square, whose side is 40 Perches in length, shall contain 10 Acres in the Superficies, and it be required to know how many Acres the Square should contain, whose side is 60 Perches.

Here if I took 10 out of the Line of Superficies, and put it over in 40, in the Lines of Lines, his Parallel between 60 and 60, measured in the Line of Superficies, would be 22 ½, and such is the number of Acres required. For Squares do hold in a duplicated Proportion of their sides; wherefore when the Proportion of their sides is as 4 to 6, and 4 multiplied into 4 become 16, and 6 multiplied into 6 become 36, the Proportion of their Squares shall be as 16 to 36, and such is the Proportion of 10 to 22 ½.

If a Field measured with a Statute Perch of 16 ½ foot, shall contain 288 Acres, and it be required to know how many Acres it would contain if it were measured with a Woodland Perch of 18 foot.

Here because the Proportional is reciprocal, if I took 288 out of the Line of Superficies, and put it over in 18 in the Lines of Lines, his Parallel between 16 ½ and 16 ½ measured in the Line of Superficies, would be 242; and such is the Number of Acres required.

For seeing the Proportion of the Sides is as 16 ½ to 18, or in lesser Numbers as 11 to 12, and that 11 multiplied into 11 become 121, and 12 into 12 become 144, the Proportion of these Superficies shall be as 121 to 144, and so have 288 to 242, in reciprocal Proportion.

On the contrary, if a question be proposed concerning the Side of a Superficies, the two Numbers of the first Denomination must be reckoned in the Lines of Superficies, and the Sector opened in the terms of the first Number to the quantity of a Line, out of the Line of Lines or some Scale of equal parts, resembling the second Number; so his Parallel taken between the terms of the third Number being measured in the

ſame Scale with the ſecond Number, ſhall give the fourth Number required.

As if a Field contained 288 Acres when it was meaſured with a Statute Perch of 16 ½, and being meaſured with another Perch, was found to contain 242 Acres, it were required to know what was the length of the Perch with which it was ſo meaſured.

Here becauſe the Proportion is reciprocal, if I took 16 ½ out of the the Line of Lines, and put it over in 242 in the Lines of Superficies, his Parallel between 288 and 288, being meaſured in the Line of Lines, would be 18, and ſuch is the length of the Perch (in Feet) wherewith the Field was laſt meaſured.

For ſeeing the Proportion of the Acres is as 288 unto 242, or in the leaſt Numbers, as 144 to 121, and that the Root of 144 is 12, and the Root of 121 is 11, the Proportion of Roots, and conſequently of the Perches, ſhall be as 12 to 11, and ſo are 16 ½ to 18 in reciprocal Proportion.

If 360 men were to be ſet in form of a long Square, whoſe Sides ſhall have the Proportion of 5 to 8; and it were required to know the Number of men to be placed in front and file: If the Sides were only 5 and 8, there ſhould be but 40 men; but there are 360: therefore, working as before, I find that,

As 40 *to the Square of* 5:
So 360 *to the Square of* 15.

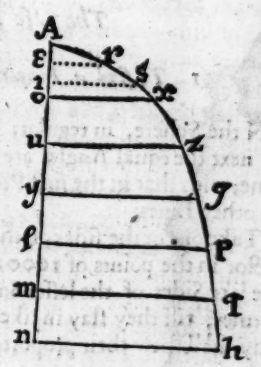
As 40 *to the Square of* 8:
So 360 *to the Square of* 24.
and ſo 15 and 24 are the Sides required.

If 1000 men were lodged in a ſquare ground whoſe Side were 60 paces, and it were required to know the Side of the Square wherein 5000 might be ſo lodged, here working as before, I ſhould find that,

As 1000, *are to the Square of* 60:
So 5000 *to the Square of* 134.
And ſuch, very near, is the Number of paces required.

11. *How to deſcribe a Parabola, by help of the Line of Lines and Superficies.*

UPon A n as the Diameter, prick down, by the Line of Lines, the equal Parts A o, A u, A y, A l, A m, A n, *&c.* and from theſe Points raiſe the Perpendiculars o x, u z, y g, l p, m q, n h, *&c.* And upon the Perpendicular o x, aſſume the Point x, and open the Sector in the Line of Superficies, ſo that o x (being the firſt Perpendicular) may fall in with the Points 1... 1 (the firſt of the Line of Superficies:) Then if you take off from the ſame Line 2..2, you ſhall prick down u z, and 3 ... 3 gives y g; and 4 ... 4, l p; 5 ... 5, m q; 6 ... 6, n h; *&c.*



Or, you may begin your work from n h, which (becauſe it is the ſixth Perpendicular) take from n to h, the Point aſſumed, and ſet that length in the Line of Superficies from 6 to 6, ſo may you prick down the other Points correſpondently.

Through theſe Points h, q, p, g, with an even hand draw the Parabola.

And here note, that Parabola's may be deſcribed of infinite Varieties, according to the Cones from whence they are taken, yet keeping all one and the ſame length.

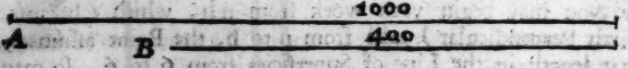
CHAP. IV.

The Use of the Lines of Solids.

1. *To find a Proportion between two or more like Solids.*

IN the Sphere, in regular, parallel, and other like bodies, whose Sides next the equal Angles are proportional, the work is in a manner the same, with that in the first Proposition of Superficies, but that it is wrought on other Lines.

Take one of the sides of the greater Solid, and according to it open the Sector in the points of 1000 and 1000, in the Lines of Solids, then take the like Sides of the lesser Solids severally, and carry them parallel to the former, till they stay in like points, so the number of points wherein they stay shall shew their proportion to 1000.



Let A and B be the like Sides of like Solids, either the Diameters or Semidiameters of two Spheres, or the sides of two Cubes or other like. First I take the side A, and to it open the Sector in the points of 1000, then keeping the Sector at this Angle, I enter the lesser Side B parallel to the former, and find it to cross the Line of Solids in the points of 400, and such is the Proportion between the Solids required, which in lesser Number is, as 5 to 2.

This Proposition might have been wrought by 60, or any other Number that admits several Divisions.

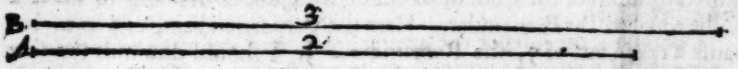
It may also be wrought without opening the Sector, for if the sides of the Solids given be applied to the Lines of Solids, beginning always at the Center of the Sector, there will be such Proportion between them, as between the Numbers of parts whereon they fall.

2. To

2. *To augment a Solid in a given Proportion.*
3. *To diminish a Solid in a given Proportion.*

TAke the side of the Solid given, and to it open the Sector, in the points of the Number given: then keeping the Sector at that Angle, the parallel Distance between the points of the Number required, shall give the like Side of the Solid required.

If it be a parallelopipedon or some irregular Solid, the other like Sides may be found out in the same manner, and with them the Solids required, may be made up with the same Angles.



Let A be the side of a Cube, to be augmented in the Proportion of 2 to 3. First, I take the side A, and put it over in the Lines of Solids in 2 and 2, so the Parallel between 3 and 3, doth give me the side B, on which if I make a Cube, it will have such Proportion to the Cube of A, as 3 to 2.

In like manner, if B were the Diameter of a Sphere, to be diminished in the proportion of 3 to 2. I would take out B, and put it over in the Lines of Solids, in 3 and 3, so the Parallel between 2 and 2, would give me A: to which Diameter if I should make a Sphere, it would be less than the Sphere, whose Diameter is B, in such proportion as 2 is less than 3.

Here also for variety of work, may the like caution be observed to that which we gave in the third Proposition of Lines.

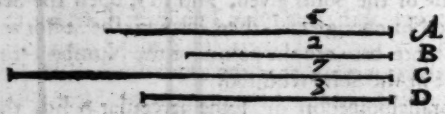
4. *To add one like Solid to another.*
5. *To subtract one like Solid from another.*

FIrst the Proportion between the sides of the like Solids given, is to be found by the first Proposition of Solids: then add or subtract those Proportions, and accordingly augment or diminish by the former Proposition.

As if A and B were the sides of two Cubes, and it were required to make a third Cube equal to them both: first the Proportion between the sides A and B, would be found to be as 100 to 40, or in lesser terms as

5 to

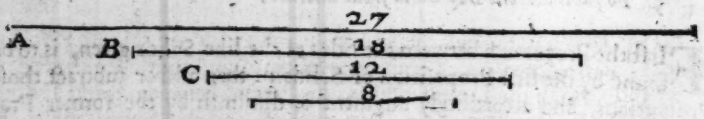
5 to 2: then because 5 and 2 being added do make 7, I augment the side A, in the proportion of 5 to 7, and produce the side C, on which if I make a Cube, it will be equal to both the Cubes of A and B, which was required.



In like manner A and B being the sides of two Cubes, if it were required to subtract the Cube of B out of the Cube of A, and to make a Cube equal to the Remainder. Here the Proportion being as 5 to 2, because 2 taken out of 5, the Remainder is 3, I should diminish the side A in the proportion of 5 to 2, and so I should have the side D, on which if I make a Cube, it will be equal to the Remainder, when the Cube of B is taken out of the Cube of A, that is, the two Cubes made upon B and D shall be equal to the first Cube made upon the side A.

6. *To find two mean proportional Lines between two extreme Lines given.*

FIrst I find what Proportion is between the two extreme Lines given, as they are Lines, by the fifth Proposition of Lines, then open the Sector in the Lines of Solids, to the quantity of the former Extreme, and a Parallel between the points of the number belonging to the other Extreme, shall be that mean Proportional, which is next the former Extreme. This done, open the Sector again to this mean Proportional in the points of the former Extreme, and the parallel Distance between the points of the latter Extreme, shall be the other mean Proportional required.



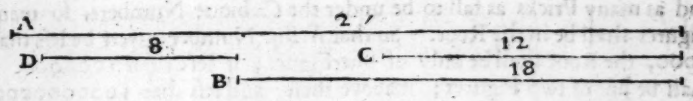
Let the two extreme Lines given be A and D, the Proportion between them, as they are Lines, will be found to be as 27 to 8. Wherefore I take the Line A, and put it over in the Lines of Solids between 27 and 27, and

27, and keeping the Sector at this Angle, his Parallel between 8 and 8, doth give me B the mean Proportional next unto A. Then put I over this Line B, between the aforeſaid 27 and 27, and this Parallel between 8 and 8 doth give me the Line C, the other mean Proportional which was required.

Again, for proof of the operation I put over this Line C in the aforeſaid 27 and 27, and his Parallel between 8 and 8 doth give me the very Line D: whereby it is plain that the e four Lines do hold in continual Proportion; and ſo B and C are found to be the Mean Proportionals between A and D the Extreme given.

7. To find two mean proportional Numbers between two extreme Numbers given.

FIrſt reckon the Numbers given on both ſides of the Lines of Solids, beginning from the Center, and marking the terms whereto they extend: then take a Line out of the Line of Lines, or any other Scale of equal parts reſembling the former of thoſe Numbers, and put it over in the Lines of Solids, between the points of his like Number, and a Parallel between the points belonging to the other Extreme, meaſured in the Scale from whence the other Parallel was taken, ſhall give that mean proportional Number which is next the former Extreme. This done open the Sector again to this mean Proportional in the Points of the former Extreme, and the parallel Diſtance between the points of the latter Extreme, meaſured in the ſame Scale as before, ſhall there ſhew the other mean Proportional required.



Let the two extreme Numbers given be 27 and 8, if I ſhall take the Line A, reſembling 27 in a Scale of equal parts, and to it open the Sector in 27 and 27, in the Line of Solids, his Parallel between 8 and 8 doth give me B, for his next mean Proportional, and this meaſured in the former Scale doth extend to 18. Then put I over this Line B, between the foreſaid 27 and 27, and his Parallel between 8 and 8 doth give me C, for the other mean Proportional, and this meaſured in the former

Scale

Scale doth extend to 12. Again, for proof of my work, I put over this Line C, between 27 and 27, as before, and his Parallel between 8 and 8 doth give me D, which measured in the former Scale doth extend to 8, which was the latter extreme Number given; whereby it is plain, that these four Numbers do hold in continual Proportion: and therefore 18 and 12 are Mean Proportionals between 27 and 8, which was required.

If you suppose Pricks under the Number given as in Arithmetical Extraction, and that last Prick to the left hand shall fall under the last figure, as in 1728, the unite will be left placed at 1, in the middle of the Line, and the Root, Square and Cube will all fall forward toward the end of the Line.

If the last Prick shall fall under the last Figure but one, as in 17280; the unite may be placed at 1, in the beginning of the Line, and the Cube in the second length: or the unite may be placed at 10, in the end of the Line, and the Cube in the first length.

But if the last Prick shall fall on the last Figure but two, as in 172800; then, place the unite always at 10 in the end of the Line: so the Root, Square and Cube will all fall backward and be found in the second length.

8. *To find the Cubique Root of a Number.*
9. *The Root being given to find the Cube Number of that Root.*

IN the Extraction of a Cubique Root, it is usual to set Pricks under the first Figure, the fourth, the seventh and tenth, and so forward omitting two, and pricking the third from the right hand toward the left; and as many Pricks as fall to be under the Cubique Numbers, so many Figures shall be in the Root. So that if the Number given be less than 1000, the Root shall be only of one Figure; if less than 1000000, it shall be but of two Figures; if above these, and less than 1000000000, it shall be but three Figures, *&c.* whereupon the Lines of Solids are divided, first into 1000 parts, and if the Numbers given be greater than 1000 the first Division (which before did signifie only one) must signifie 1000, and the whole Line shall be 1000000: if yet the Number given be greater than 1000000, the first Division must now signifie 1000000, and the whole Line be esteemed at 1000000000 parts, and if these be too little to express the Numbers given, as oft as we have recourse to the beginning, the whole Line shall increase itself a thousand times.

By

By theſe means, if the laſt Prick, to the left hand, ſhall fall under the laſt Figure, the Number given ſhall be reckoned at the beginning of the Lines of Solids from 1 to 10, and the firſt Figure of the Root ſhall be always either 1 or 2. If the laſt Prick ſhall fall under the laſt Figure but one, then the Number given ſhall be reckoned in the middle of the Line of Solids, between 10 and 100, and the firſt Figure of the Root ſhall be always either 2, or 3, or 4. But if the laſt Prick ſhall fall under the laſt Figure but two, then the Number given ſhall be reckoned at the end of the Line of Solids, between 100 and 1000.

This being conſidered, when a Number is given, and the Cubique Root required, ſet one Foot of the Compaſſes in the Center of the Sector, extend the other in the Line of Solids to the Points of the Number given: For this Diſtance applied to one of the Lines of Lines, ſhall ſhew what the Cubique Root is, without opening the Sector.

So the neareſt Root of 8490000, is about 204.
The neareſt Root of 84900000, is about 439.
The neareſt Root of 849000000, is about 947.

On the contrary, a Number may be cubed, if firſt we extend the Compaſſes to the Number given, in the Line of Lines, and then apply the Diſtance to the Lines of Solids, as may appear by the former Examples.

10. *Three Numbers being given, to find a fourth in a triplicated Proportion.*

AS like Superficies do hold in a duplicated Proportion, ſo like Solids in a triplicated Proportion of their homologal Sides: and therefore the ſame Work is to be obſerved here on the Lines of Solids, as before in the Lines of Superficies, as may appear by theſe two Examples.

If a Cube whoſe ſide is 4 inches, ſhall be 7 pound weight, and if it be required to know the weight of a Cube whoſe ſides is 7 inches; here the Proportion would be,

As 4 *are to a Cube of* 7:
So 7 *to a Cube of* 37 ½.

And if I took 7 out of the Lines of Solids, and put it over in 4 and 4, in the Lines of Lines, his Parallel between 7 and 7, measured in the Lines of Solids, would be 37 ½; and such is the weight required.

If a Bullet of 27 pound weight, have a Diameter of 6 inches, and it be required to know the Diameter of the like Bullet, whose weight is 125 pounds; here the Proportion would be,

As the Cubique Root of 27, *is unto* 6,
So the Cubique Root of 125, *is unto* 10.

And if I took 6 out of the Line of Lines, and put it over in 27, and 27 of the Lines of Solids, his Parallel between 125 and 125 measured in the Line of Lines, would be 10; and such is the length of the Diameter required.

The End of the first Book.

THE

THE SECOND BOOK OF THE SECTOR,

Containing the Uſe of the Circular Lines.

CHAP. I.

Of the Nature of Sines, Chords, Tangents, and Secants, fit to be known before-hand, in reference to right-lined Triangles.

IN the Canon of Triangles, a Circle is commonly divided into 360 Degrees, each Degree into 60 Minutes, each Minute into 60 Seconds.

A Semicircle therefore is an Ark of 180 *gr.*

A Quadrant is an Ark of 90 *gr.*

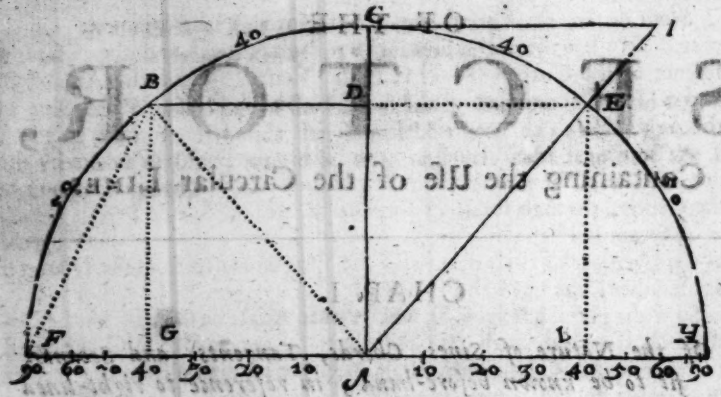
The meaſure of an Angle is the Ark of a Circle deſcribed out of the angular point, intercepted between the Sides ſufficiently produced.

So the meaſure of a right Angle is always an Ark of 90 *gr.* and in this Example the meaſure of the Angle B A D in the following Figure, is the Ark B C of 40 *gr.* the meaſure of the Angle B A G, is the Ark B F of 50 *gr.*

The Complement of an Ark or of an Angle doth commonly ſignifie the Ark which the given Ark doth want of 90 *gr.* and ſo the Ark B F is the Complement of the Ark B C, and the Angle B A F, whoſe

measure is B F, is the Complement of the Angle B A C; and on the contrary,

The Complement of an Ark or Angle in regard of a Semicircle, is that Ark which the given Ark wanted to make up 180 *gr.* and so the Angle E A H is the Complement of the Angle E A F, as the Ark E H is the Complement of the Ark F E, in which the Ark C E is the excess above the Quadrant.



The Proportions which these Arks (being the measures of Angles) have to the Sides of a Triangle, cannot be certain, unless that which is crooked be brought to a streight Line, and that may be done by the application of *Chords*, *Right Sines*, *Versed Sines*, *Tangents* and *Secants* to the Semidiameter of a Circle.

A Chord is a right Line subtending an Ark: so B E is the Chord of the Ark B C E, and B F a Chord of the Ark B 40 F.

A right Sine is half the Chord of the double Ark, *viz.* the right Line which falleth perpendicularly from the one Extreme of the given Ark, upon the Diameter drawn to the other Extreme of the said Ark.

So if the given Ark be B C, or the given Angle be B A C, let the Diameter be drawn through the Center A unto C, and a Perpendicular B D be let down from the Extreme B upon A C, this Perpendicular B D shall be the right Sine both of the Ark B C, and also of the Angle B A C:

B A C: and it is alſo the half of the Chord B E, ſubtending the Ark B C E, which is double to the given Ark B C. In like manner, the Semidiameter F A, is the right Sine of the Ark F C, and of the right Angle F A C; for it falleth perpendicularly upon A C, and it is the half of the Chord F H.

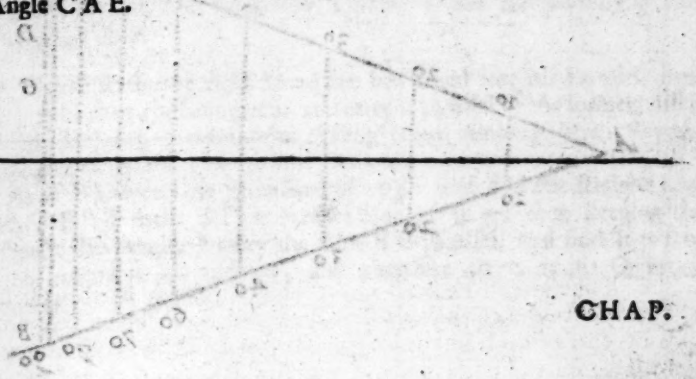
This whole Sine of 90 *gr.* is hereafter called *Radius*, but the other Sines take their Denomination from the Degrees and Minutes of their Arks.

Sinus verſus, the Verſed Sine is a Segment of the Diameter, intercepted between the right Sine of the ſame Ark, and the Circumference of the Circle. So D C is the Verſed Sine of the Ark C B, and G F the Verſed Sine of the Ark B F, and G H the Verſed Sine of the Ark B H.

A Tangent is a right Line perpendicular to the Diameter drawn by the one Extreme of the given Ark, and terminated by the Secant drawn from the Center, through the other Extreme of the ſaid Ark.

A Secant is a right Line drawn from the Center, through one Extreme of the given Ark, till it meet with the Tangent raiſed from the Diameter at the other Extreme of the ſaid Ark.

So if the given Ark be C E, or the given Angle be C A E, let the Diameter be drawn through the Center A to C, and in C to A C, be raiſed a Perpendicular C I. Then let another Line be drawn from the Center A, through E, till it meet with the Perpendicular C I in I; the Line C I is a Tangent, and A I is the Secant both of the Ark C E, and of the Angle C A E.



CHAP.

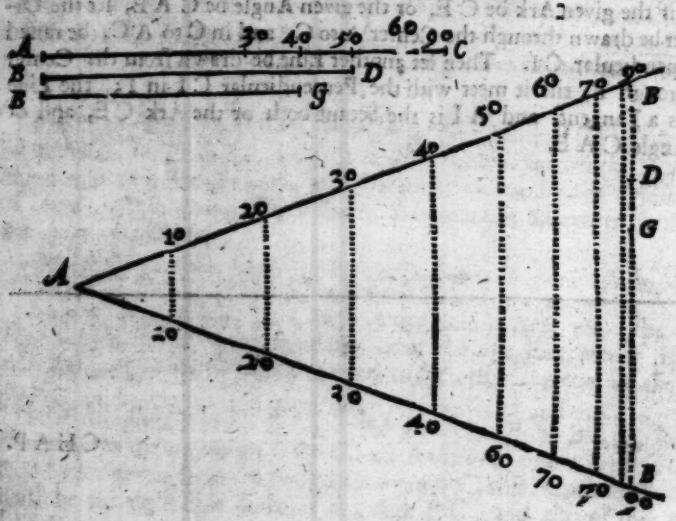
CHAP. II.

Of the general Use of Sines and Tangents.

1. *The Radius being known, to find the right Sine of any Ark or Angle.*

IF the Radius of the Circle given be equal to the lateral Radius, that is to the whole Line of Sines on the Sector, there needs no farther work, but to take the other Sines also out of the Side of the Sector. But if it be either greater or lesser, then let it be made a parallel Radius, by applying it over in the Lines of Sines, between 90 and 90, so the Parallel taken from the like lateral Sines, shall be the Sine required.

As if the given Radius be A C, and it were required to find the Sine of 56 *gr.* and his Complement agreeable to that Radius.



Let

Let A B, A B repreſent the Lines of Sines on the Sector, and let B B the Diſtance between 90 and 90, be equal to the given Radius A C. Here the Lines A 40, A 50, A 90 may be called the lateral Sines of 40, 50, and 90, in regard of their place on the ſide of the Sector. The Lines between 40 and 40, between 50 and 50, between 90 and 90, may be called the parallel Sines of 40, 50, and 90, in regard they are parallel one to the other. The whole Sine of 90 *gr.* here ſtanding for the Semidiameter of the Circle, may be called the Radius. And therefore if A C be put over in the Line of Sines in 90 and 90, and ſo made a Parallel Radius, his parallel Sine between 50 and 50 ſhall be B D, the Sine of 50 required. And becauſe 50 taken out of 90, the Complement is 40, his parallel Sines between 40 and 40 ſhall be B G, the Sine of the Complement which was required.

2. *The right Sine of any Ark being given to find the Radius.*

TUrn the Sine given into a parallel Sine, and his parallel Radius ſhall be the Radius required.

As if B D were the given Sine of 50 *gr.* and it were required to find the Radius, let B D be made a parallel Sine of 50 *gr.* by applying it over in the Lines of Sines between 50 and 50, ſo his parallel Radius between 90 and 90 ſhall be A C, the Radius required.

3. *The Radius of a Circle, or the right Sine of any Ark, being given, and a ſtreight Line reſembling a Sine, to find the quantity of that unknown Sine.*

LEt the Radius or right Sine given be turned into his Parallel, then take the right Line given, and carry it parallel to the former, till it ſtay in like Sines, ſo the number of Degrees and minutes where it ſtayeth, ſhall give the quantity of the Sine required.

As if B D were the given Sine of 50 *gr.* and B G the ſtreight Line given, firſt I make B D a parallel Sine of 50 *gr.* then keeping the Sector at this Angle, I carry the Line B G parallel, and find it to ſtay in no other but 40 and 40, and therefore 40 *gr.* is his Quantity required.

4. *The*

4. The Radius or any right Sine being given, to find the Verſed Sine of any Ark.

IF the Ark, whoſe Verſed Sine is required, be leſs than the Quadrant, take the Sine of the Complement out of the Radius, and the Remainder ſhall be the *Sinus Verſus*, the Verſed Sine of that Ark.

As if A B being the lateral Radius, it were required to find the Verſed Sine of 40 *gr.* here the Sine of the Complement is A 50, and therefore B 50 is the Verſed Sine required. Or if I reckon from B at the end of the Sector, toward the Center, the Diſtance from 90 to 80 is the Verſed Sine of 10 *gr.* from 90 to 70, the Verſed Sine of 20 *gr.* from 90 to 60 is the Verſed Sine of 30 *gr.* and ſo in the reſt.

If A D be the given Sine of 50 *gr.* and it be required to find the Verſed Sine of 50 *gr.* here becauſe A D is unequal to the lateral Sine of 50 *gr.* I make it a Parallel. And firſt I find the Radius A C, then the Sine of the Complement A 40, which being taken out of A C, leaveth C 40, for the Verſed Sine of 50 *gr.* which was required.

But if the Ark whoſe Verſed Sine is required, be greater than the Quadrant, his Verſed Sine alſo is greater than the Radius, by the right Line of his exceſs above 90 *gr.*

As if A C being the Radius given, it were required to find the Verſed Sine of 130 *gr.* here the exceſs above 90 *gr.* is 40 *gr.* and therefore the Verſed Sine required is equal to the Radius A C and A 40, both being ſet together.

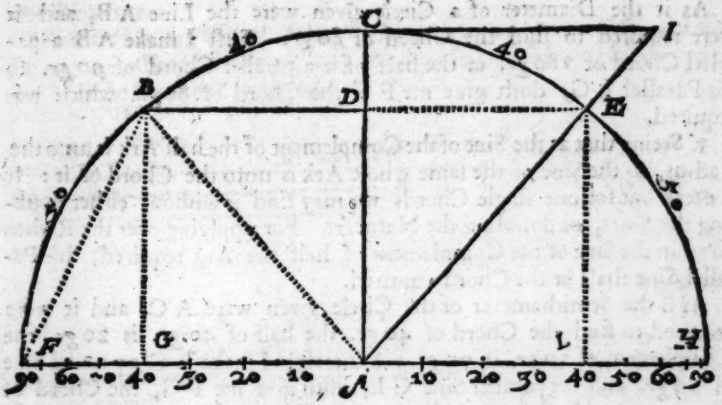
5. The Diameter or Radius being given, to find the Chords of every Ark.

THe Sines may be fitted many ways to ſerve for Chords, 1. A Sine being the half of the double Ark, if the Sine be doubled; it giveth the Chord of the double Ark, a Sine of 10 *gr.* doubled giveth a Chord of 20 *gr.* and a Sine of 25 *gr.* being doubled giveth a Chord of 50 *gr.* and ſo in the reſt; As here B D, the Sine of B C, an Ark of 40 *gr.* being doubled, giveth B E the Chord of B C E, which is an Ark of 80 *gr.* Wherefore if the Radius of the Circle given be equal to the lateral Radius, let the Sector be opened near unto his length, ſo that both the Lines of Sines may make but one direct Line: ſo the Diſtance on the Sines between 10 and 10 ſhall be a Chord of 20, the Diſtance between

20 and

30 and 20 ſhall be a Chord of 40, and the Diſtance between 30 and 30, ſhall be a Chord of 60, and ſo in the reſt.

2. Becauſe a Sine is the half of the Chord of the double Ark, the Proportion holdeth.



As the Diameter F H unto the Radius A H, ſo the Chord B E unto the Sine D E, or the Chord G L unto the Sine A L, and then if the Radius A H be put for the Diameter, which is a Chord of 180 *gr.* the Sine D E or A L, ſhall ſerve for a Chord of 80 *gr.* and the Semiradius which is the Sine of 30 *gr.* ſhall ſerve for a Chord of 60 *gr.* and go for the Semidiameter of a Circle, and ſo in the reſt. So that by theſe means we ſhall not need to double the Lines of Sines as before, but only to double the Numbers. And to this purpoſe I have ſubdivided each degree of the Sines into two, that ſo they might ſhew how far the half degrees do reach in the Sines, and yet ſtand for whole degrees when they are uſed as Chords.

Wherefore if the Radius of the Circle given be equal to the lateral Semiradius (the Sine of 30 *gr.* and Chord of 60 *gr.*) there needs no farther work, then to take the Sine of 10 *gr.* for a Chord of 20 *gr.* and a Sine of 15 *gr.* for a Chord of 30 *gr.* *&c.*

But if the Radius of the Circle given be either greater or leſſer than the lateral Semiradius, take the Diameter of it, and make it a Parallel Chord of 180 *gr.* by applying it over the Lines of Sines between 90

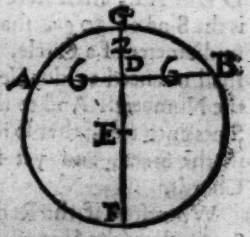
and 90, or take the Radius or Semidiameter, which is equal to the Chord of 60 *gr.* and make it a parallel Radius of 60 *gr.* by applying it over in the Sines of 30 and 30, and keep the Sector at this Angle. The Parallels taken from the lateral Chords ſhall be the Chords required.

As if the Diameter of a Circle given were the Line A B, and it were required to find the Chord of 80 *gr.* Firſt I make A B a parallel Chord of 180 *gr.* or the half of it a parallel Chord of 90 *gr.* ſo his Parallel L G, doth give me F G the Chord of 80 *gr.* which was required.

3. Seeing that as the Sine of the Complement of the half Ark is unto the Radius, ſo the Sine of the ſame whole Ark is unto the Chord of it: If we ſeek but for one ſingle Chord, we may find it without either doubling the Sines, or doubling the Number. For applying over the Radius given in the Sine of the Complement of half the Ark required, his Parallel Sine ſhall be the Chord required.

As if the Semidiameter of the Circle given were A C, and it were required to find the Chord of 40 *gr.* the half of 40 *gr.* is 20 *gr.* the Complement of 20 *gr.* is 70 *gr.* Wherefore I make A C a parallel Sine of 70 *gr.* and his parallel Sine G L, doth give me F G, the Chord of 40 *gr.* agreeable to the Semidiameter A C.

Having two right Lines reſembling the Chord and Verſed Sine, to find the Diameter and Radius.



Let the two right Lines given be A B, reſembling the Chord, G D the Verſed Sine of a Circle, whoſe Arch A G B is unknown, and and let it be required to find the Diameter G F.

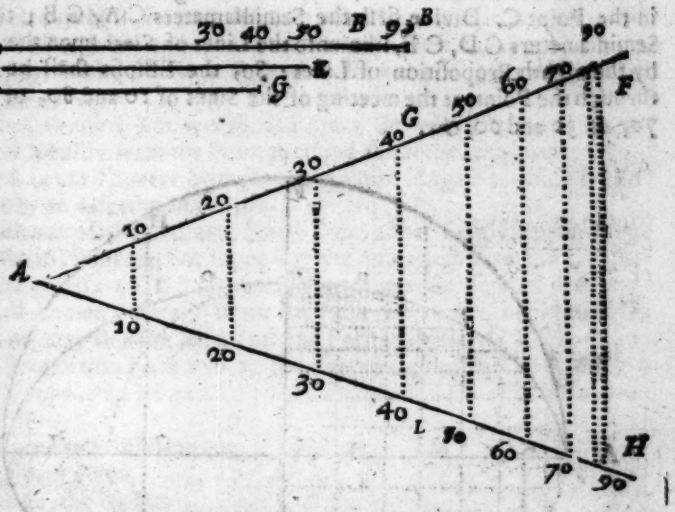
Having two Lines given, the firſt G D, the ſecond A D, the half of A B, we may find a third in continual Proportion (by the ſixth or nineth Propoſition of the Lines) and that ſhall be the Line D F (18) the Sum whereof and of G D gives the Diameter G F (20) and the half thereof is the Radius (E G).

6. The

6. The Chord of any Ark being given, to find the Diameter and Radius.

TUrn the Chord given unto a parallel Chord, and his parallel Semi-radius ſhall be the Semidiameter, and the parallel Radius ſhall be the Diameter.

As if F G be the Chord of 80 *gr.* I put this over in G and L, the Sine of 40, and Chord of 80 *gr.* and the parallel Chord of 180 *gr.* giveth me A B the Diameter required.



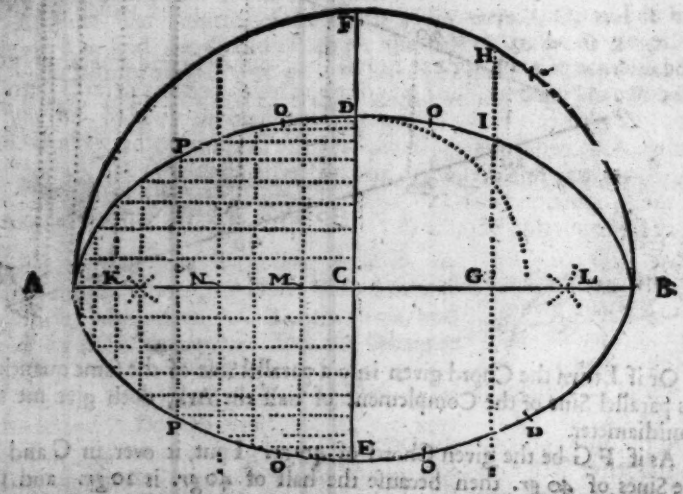
Or if I turn the Chord given into a parallel Sine of the ſame quantity, his parallel Sine of the Complement of half the Ark, doth give me the Semidiameter.

As if F G be the given Chord of 40 *gr.* I put it over in G and L, the Sines of 40 *gr.* then becauſe the half of 40 *gr.* is 20 *gr.* and the Complement of 20 *gr.* is 70 *gr.* I take out the parallel Sine of 70 *gr.* and it giveth me A B for the Semidiameter, agreable to that Chord of 40 *gr.*

Having the Diameter of an Ellipsis to describe the same upon a plain.

If each Semidiameter be divided, in such sort, as the Line of *Sines* is divided upon the *Sector*, and right Lines drawn through each division Perpendicular to those Semidiameters like unto Sines; The Points where the Sines drawn through the one Semidiameter do meet the Sines of the Complement drawn through the other Semidiameter, shall be the Points through which the Ellipsis is to be drawn.

Let the Diameters be A B, D E, one crossing the middle of the other in the Point C. Divide first the Semidiameters C A, C B; then the Semidiameters C D, C E, like unto the Lines of *Sines* upon the *Sector*, by the eighth Proposition of Lines: So, the Ellipsis shall be drawn through the Points at the meeting of the Sines of 10 and 80, of 20 and 70, of 30 and 60, &c.



Or (without the help of the Line of Sines) we may draw the Circle A F B upon the Center C, and Semidiameter A C, for so, crossing the Diameter A B with several Perpendicular Lines continued unto the Circumference of the Circle, if we divide these Perpendiculars on either

either side of the Diameter in such sort as the greater Semidiameter C F is divided by the lesser, in the Point D, and draw a Line winding through all those Points, the Line so drawn shall be the Ellipsis.

Or (without the help of the *Sector*) we may with the Radius A C, upon the Centers D and E, describe two occult Arches meeting in the Points K and L. Then taking between C and K, any Number of of Points M N, we may from the Centers K and L, with the Semidiameter M B describe four occult Arches; and with the Radius A M, and the same Centers K and L, cross them again with other four Arches in the Points at O. In like manner, from the same Centers K and L, with the Radius N B, we may describe other four occult Arches; and with the Radius A N, and the former Centers cross them again, with four Arches in the Points at P, and so draw the Ellipsis through the Points O P, *&c.*

This is (in effect) as we should tye a thread about A and L, and then draw it easily from the Point A round about the two former Centers K and L, until it were brought to the Point *A* again: which is also an easie way to describe an Ellipsis.

The distance of these former Points from either Semidiameter may be set down in Numbers. For supposing the lesser Semidiameter C D, to be 10, the greater (C B) to be 16, (or otherwise divided into any Number of known Points,) If we have the proportion between C G and C B, we may find the length of the Perpendicular G I.

If the Proportion be as 1 to 2, the Perpendicular will be 8. 66.

If the Proportion be as 2 to 3, the Perpendicular will be about 7. 45.

As the greater Semidiameter	C B
to the part given	C G.
So 100000, the Radius	C B
to the Sine of	C G
whose Complement is	G H
As the Radius	C F
to the Sine of the Complement	G H
So the lesser Semidiameter	C D
to the Perpendicular	G I

The same may also be found without knowing the Sines. For the Perpendicular G H is a mean Proportional between A G and G B: which being known

As C F unto E D, so is G H unto G I.

7. To

7. *To open the* Sector *to the quantity of any Angle given.*
8. *The* Sector *being opened, to find the quantity of the Angle.*

IT is one thing to open the Edges of the Sector to an Angle, and another thing to open the Lines on the Sector to the ſame Angle. For the Lines of *Lines* on the one ſide, and the Lines of *Sines* on the other ſide, do make an Angle of 2 *gr.* when the Sector is cloſe ſhut, and the Edges do make no Angle at all. So likewiſe the Lines of *Superficies* and the Lines of *Solids* do make an Angle of 10 *gr.* which are to be allowed to the Edges.

The Lines of *Lines* may be opened to a right Angle, if the whole Line of 100 parts be applied over in 80 and 60.

The Line of *Sines* may be opened to a right Angle, if the large Secant of 45 *gr.* be applied over in the Sines of 90 *gr.* or if the Sine of 90 *gr.* be applied over in the Sines of 45 *gr.* or if the Sine of 45 *gr.* be applied over in the Sines of 30 *gr.*

If it be required to open thoſe Lines to any other Angle, take out the Chord thereof, and apply it over in the *Semiradius*, and thoſe Lines ſhall be opened to that Angle.

As if it were required to open the Sector in the Lines of *Sines* to an Angle of 40 *gr.* take out the Chord of 40 *gr.* and to it open the Sector in the Chord of 60 *gr.* ſo ſhall the Lines of *Sines* be opened to the Angle required. Or if the ſame Chord of 40 *gr.* be applied over between 50, and 50, in the Line of *Lines*, they ſhall alſo be opened to the ſame Angle. If it be applied over in 25 of the Lines of *Superficies*, or 125 in the Lines of *Solids*, they alſo ſhall be opened to the ſame Angle: becauſe the Chord of 60 *gr.* or Sine of 30 *gr.* and 50 in the Lines of *Lines*, and 25 in the Lines of *Superficies*, and 125 in the *Solids*, are all of the ſame length with the *Semiradius*.

Or if the *Semiradius* by applied over between the Sine of 30 *gr.* and the Sine of the Complement of the Angle required, it will open the Lines of *Sines* to that Angle.

As if the *Semiradius* be applied over in the Sines of 30 *gr.* and the Sine of 50 *gr.* it ſhall open the Lines of *Sines* to an Angle of 40 *gr.*

On the contrary, if the *Sector* be opened to an Angle, and it be required to know the quantity thereof, open the Compaſſes to the *Semiradius*, and ſetting one foot in the Sine of 30 *gr.* turn the other toward the other Line of *Sines*, and it ſhall fall there in the Complement

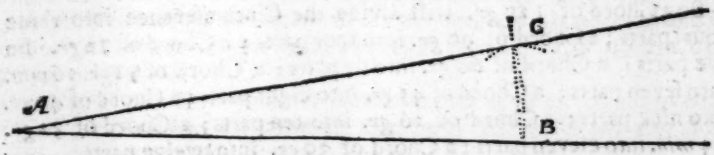
of the Angle; if it fall on 50 *gr.* the Angle is 40 *gr.* if on 60 *gr.* the Angle is 30 *gr.* &c.

Or take over the parallel Chord of 60 *gr.* and meaſure it in the lateral Chord, and it ſhall there ſhew the quantity of the Angle. As if the *Sector* being opened to an Angle, I ſhould take over the Parallel of 30 *gr.* of the Sines, and 60 *gr.* of the Chords, and meaſure it in the lateral Chords, find it to be 40 *gr.* the Angle comprehended between the Lines of *Sines* is 40 *gr.* but the Angle between the Edges of the Sector is 2 *gr.* leſs, and therefore but 38 *gr.*

9. *To find the quantity of any Angle given.*

IF out of the Angular Point, to the quantity of the *Semiradius*, be deſcribed an occult Ark that may cut both ſides of the Angle, the Chord of this Ark meaſured in the lateral Chord, ſhall give the quantity of the Angle.

Let the Angle given be B A C: firſt I take the *Semiradius* with the Compaſſes, and ſetting one foot in A, I cut the ſides of the Angle in B and C; then I take the Chord B C, and meaſure it in the lateral Chord, and I find it to be 11 *gr.* and 15 *min.* and ſuch is the quantity of he Angle given.



Or if the Ark be deſcribed out of the Angular Point at any other diſtance, let the Semidiameter be turned into a parallel Chord of 60 *gr.* then take the Chord of this Ark, and carry it Parallel, till it croſs in like Chords: ſo the place where it ſtayeth ſhall give the quantity of the Angle.

As in the former example, if I make the Semidiameter A B a parallel Chord of 60 *gr.* and then keeping the *Sector* at that Angle, carry the Chord B C parallel, till it ſtay in like Chords; I ſhall find it to ſtay in no other but 11 *gr.* 15 *min.* and ſuch is the Angle B A C.

10. *Upon*

10. *Upon a right Line, and a Point given in it, to make an Angle equal to any Angle given.*

FIrſt out of the Point given deſcribe an Ark, cutting the ſame Line: then by the 5 *Prop.* afore, find the Chord of the Angle given agreeable to the Semidiameter, and inſcribe it into this Ark: ſo a right Line drawn through the Point given, and the end of this Chord, ſhall be the ſide that makes up the Angle.

Let the right Line given be A B, and the Point given in it be *A*, and let the Angle given be 11 *gr.* 15 *min.* Here I open the Compaſſes to any Semidiameter A B, (but as oft as I may conveniently to the lateral Semiradius) and ſetting one foot in *A*, I deſcribe an occult Ark B C; then I ſeek out the Chord of 11 *gr.* 15 *min.* and taking it with the Compaſſes, I ſet one foot in B, the other croſſeth the Ark in C, by which I draw the Line A C, and it makes up the Angle required.

11. *To divide the Circumference of a Circle into any parts required.*

IF 360, the meaſure of the whole Circumference, be divided by the Number of parts required, the Quotient giveth the Chord, which being found will divide the Circumference.

So a Chord of 120 *gr.* will divide the Circumference into three equal parts; a Chord of 90 *gr.* into four parts; a Chord of 72 *gr.* into five parts; a Chord of 60 *gr.* into ſix parts; a Chord of 51 *gr.* 26 *min.* into ſeven parts; a Chord of 45 *gr.* into eight parts; a Chord of 40 *gr.* into nine parts; a Chord of 36 *gr.* into ten parts; a Chord of 32 *gr.* 44 *min.* into eleven parts; a Chord of 30 *gr.* into twelve parts.

In like manner if it be required to divide the Circumference of the Circle whoſe Semidiameter is A B, into 32; firſt I take the Semidiameter A B, and make it a parallel Chord of 60 *gr.* then becauſe 360 *gr.* being divided by 32 the Quotient will be 11 *gr.* 15 *min.* I find the parallel Chord of 11 *gr.* 15 *min.* and this will divide the Circumference into 32.

But here the parts being many, it were better to divide it firſt into fewer, and after to come over it again. As firſt to divide the Circumference into 4, and then each 4 parts into 8, or otherwiſe, as the parts may be divided.

12. *To*

12. *To divide a right Line by extreme and mean proportion.*

THe Line to be divided by extreme and mean proportion, hath the ſame proportion to his greater Segment, as in Figures inſcribed in the ſame Circle, the ſide of an *Hexagon* a figure of ſix Angles, hath to a ſide of a *Decagon* a figure of ten Angles: but the ſide of a *Hexagon* is a Chord of 60 *gr.* and the ſide of a *Decagon* is a Chord of 36 *gr.*

Let A B be the Line to be divided: if I make A B a parallel Chord of 60 *gr.* and to this Semidiameter find A C a Chord of 36 *gr.* this A C ſhall be the greater Segment, dividing the whole Line in C, by extreme and mean proportion. So that,

As A B the whole, is unto A C the greater Segment: ſo A C the greater Segment, unto C B the leſſer Segment.

Or let A C be the greater Segment given: if I make this a parallel Chord of 36 *gr.* the correſpondent Semidiameter ſhall be the whole Line A B, and the difference C B the leſſer Segment.

A C B

Or let C B be the leſſer Segment given: if I make this a parallel Chord of 36 *gr.* the correſpondent Semidiameter ſhall be the greater Segment A C, which added to C B, gives the whole Line A B.

To avoid doubling of Lines or Numbers, you may put over the whole Line in the *Sines* of 72 *gr.* and the parallel Sine of 36 *gr.* ſhall be the greater Segment.

Or if you put over the whole Line in the Sines of 54 *gr.* the parallel Sine of 30 *gr.* ſhall be the greater Segment, and the parallel Sine of 18 *gr.* ſhall be the leſſer Segment.

CHAP. III.

Of the projection of the Sphere *in Plano.*

SECT. I.

To Project the Sphere in Plano, by ſtreight Lines.

1. THe Sphere may be projected in *Plano* in ſtreight Lines, as in the *Analemma*, if the Semidiameter of the Circles given be divided in ſuch ſort as the Line of *Sines* on the Sector.

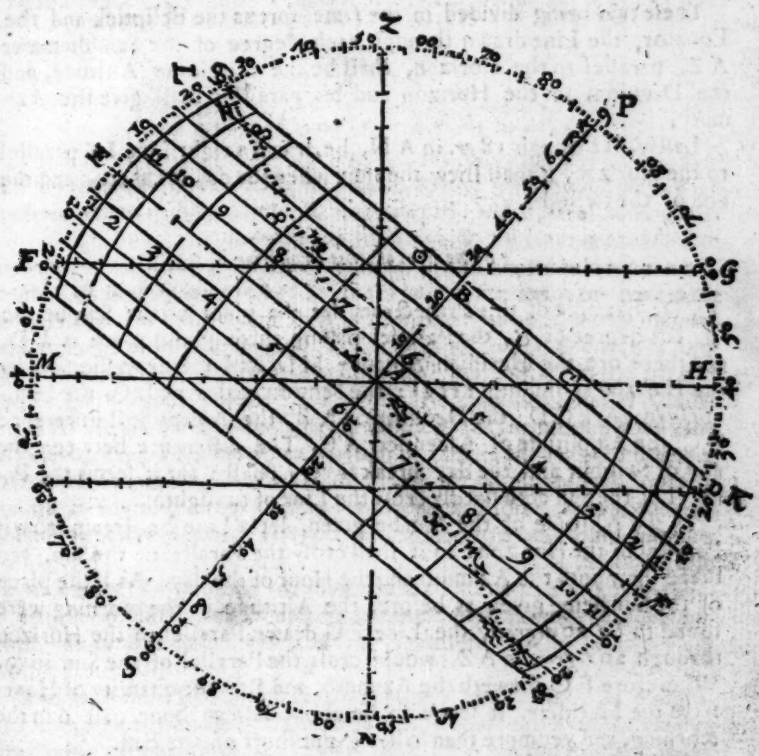
As if the Radius of the Circle given were A E, the Circle thereon deſcribed may repreſent the Plane of the general Meridian, which divided into four equal parts in E, P, Æ, S, and croſſed at right Angles with E Æ and P S, the Diameter E Æ, ſhall repreſent the Equator, and P S, the Circle of the hour of 6. And it is alſo the Axis of the World, wherin P ſtands for the North Pole, and S for the South Pole. Then may each quarter of the Meridian be divided into 90 *degrees* from the Equator towards the Poles. In which we number 23 *degrees*, 30 *min.* the greateſt declination of the Sun from E to ♋ Northwards, from Æ to ♑ Southwards, the Line drawn from ♋ to ♑ ſhall be the Ecliptick, and the Lines drawn parallel to the Equator through ♋ and ♑ ſhall be the Tropick.

Having theſe common Sections with the Plane of the Meridian, if we ſhall divide each Semidiameter of the Ecliptick into 90 *degrees*, in ſuch ſort as the *Sines* are divided on the *Sector*. The firſt 30 *degr.* from A towards ♋ ſhall ſtand for the Sign of ♈. The 30 *degr.* next following for ♉. The reſt of ♊ ♋ ♌, *&c.* in their order. So that by theſe means we have the place of the *Sun* for all times of the year.

If again we divide A P, A S, in the like ſort, and ſet thereto the Numbers 10, 20, 30, *&c.* unto 90 *degrees*, the Lines drawn through each of theſe degrees parallel to the Equator ſhall ſhew the declination of the Sun, and repreſent the Parallels of Latitude.

If farther, we divide A E, A Æ, and each of his Parallels equally in the like ſort, and then carefully draw a Line through each 15 *degr.* ſo as it makes no Angles, the Lines ſo drawn ſhall be Eliptical, and repreſent

represent the Hour-circles. The Meridian P E S, the hour of 12 at noon; that next unto it drawn through 75 *degrees* from the Center, the hours of 11 and 1, that which is drawn through 60 *degrees* from the Center, the hours of 10 and 2, *&c.*



To these we may add the months of the year, and the days of each month, placing *January* about F, *March* about E, *June* about J, *July* about K, *September* about E Æ, *December* about the Tropick of ♑: and so the rest acccording to their Declination from the Equator.

Then having respect unto the Latitude, we may number it from

E Northward unto Z, and there place the Zenith: by which, and the Center, the Line drawn Z A N, ſhall the Vertical Circle, paſſing through the Zenith and Nadir, and through the Center at A, in the Points of Eaſt and Weſt, and the Line M A H croſſing it at right Angles, ſhall repreſent the Horizon.

Theſe two being divided in the ſame ſort as the Ecliptick and the Equator, the Line drawn through each degree of the Semidiameter A Z, parallel to the Horizon, ſhall be the Circles of Altitude, and the Diviſions in the Horizon and his parallels ſhall give the Azimuth.

Laſtly, If through 18 *gr.* in A N, be drawn a right Line I K parallel to the Horizon, it ſhall ſhew the time when the day breaketh, and the end of the twilight.

Some Uſes of this Projection.

FOr Example of this Projection, let the place of the Sun be the laſt degree of ♉, the Parallel paſſing through this place is L D, and therefore the Meridian Altitude M L, and the depreſſion below the Horizon at midnight H D: the Semidiurnal Ark L C, the Seminocturnal Ark C D; the Declination A B, the Aſcentional difference B C, the Amplitude of Aſcention A C. The difference between the end of twilight and the day break is very ſmall; for it ſeems the Parallel of the Sun doth hardly croſs the Line of twilight.

If the Altitude of the Sun be given, let a Line be drawn from it Parallel to the Horizon: ſo it ſhall croſs the Parallel of the Sun, and there ſhew both the Azimuth and the Hour of the day. As if the place of the Sun being given as before, the Altitude in the morning were found to be 20 *degrees*, the Line F G drawn Parallel to the Horizon through 20 *degrees* in A Z, would croſs the Parallel of the Sun in ⊙. Wherefore F ⊙ ſheweth the Azimuth, and L ⊙ the quantity of Hours from the Meridian. It ſeems to be about half an hour paſt 6 in the morning, and yet more than half a Point ſhort of the Eaſt.

The diſtance of two places may be alſo ſhewed by this Projection, their Latitudes being known, and their difference of Longitude.

For ſuppoſe a place in the Eaſt of *Arabia*, having 20 *degrees* of North Latitude, whoſe difference of Longitude from *London*, is found by an Eclipſe to be 5 Hours, ½. Let Z be the Zenith of *London*, the Parallel of Latitude for that other place muſt be L D, in which the difference

difference of Longitude is L ☉. Wherefore ⊙ repreſenting the ſite of that place, I draw through ☉ a Parallel to the Horizon M H, croſſing the Vertical A Z near about 70 *degrees* from the Zenith, which multiplied by 20, ſheweth the diſtance of *London*, and that place to be 1400 Leagues. Or multiplied by 60, to be 4200 miles.

Sect. II.

To project the Sphere in Plano, by Circular Lines.

2. THe Sphere may be projected in *Plano* by Circular Lines, as in the general Aſtrolabe of *Gemma Friſius*, by the help of the Tangent on the ſide of the *Sector*.

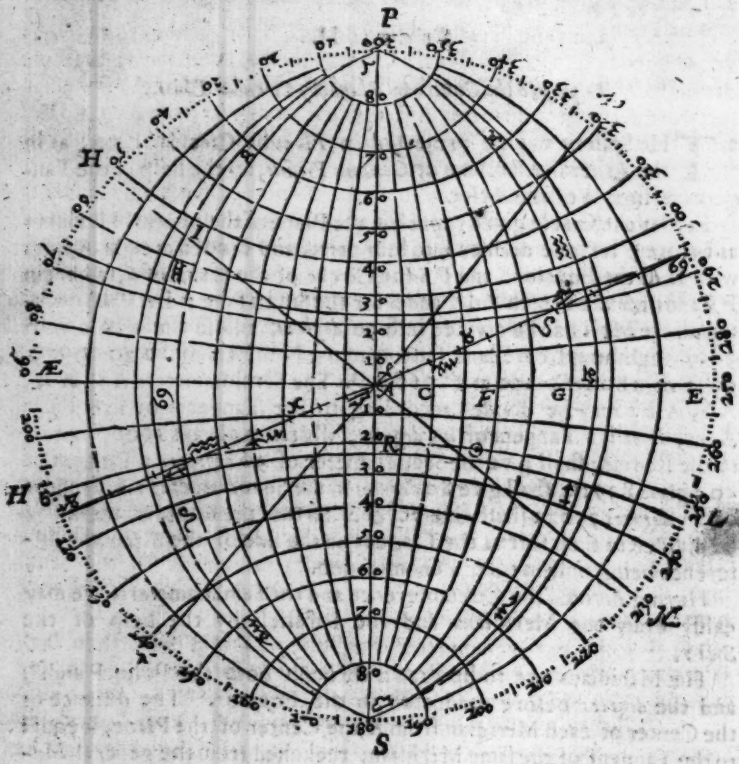
For let the Circle given repreſent the Plane of the general Meridian as before; let it be divided into four parts, and croſſed at right Angles with E Æ the Equator, and P S the Circle of the Hour of 6, wherein P ſtands for the North Pole, and S for the South Pole: Let each quarter of the Meridian be divided into 90 *degrees*, and ſo the whole into 360, beginning from P, and ſetting to the Numbers of 10, 20, 30 *&c.* 90 at Æ, 180 at S, 270 at E, 360 at P. The Semidiameters A P, A Æ, A S, A E, may be divided according to the Tangents of half their Arks, that is a Tangent of 45 *degrees*, which is always 10000, equal to the Radius, ſhall give the Semidiameter of 90 *degrees*, a Tangent of 40 *degrees* 83910, ſhall give 80 *degrees* in the Semidiameter: a Tangent of 35 *degrees* 70021 ſhall give 70, *&c.* So that the Semidiameters may be divided in ſuch ſort as the Tangent on the ſide of the *Sector*, the difference being only in their denomination.

Having divided the Circumference and the Semidiameters, we may eaſily draw the Meridians and the Parallels by the help of the *Sector*.

The Meridians are to be drawn through both the Poles P and S, and the *degrees* before graduated in the Equator. The diſtance of the Center of each Meridian from A, the Center of the Plane, is equal to the Tangent of the ſame Meridian, reckoned from the general Meridian P Æ S E, and the Semidiameter equal to the Secant of the ſame *degree*.

As for example, If I ſhould draw the Meridian P B S, which is the tenth from P Æ S, the Tangent of 10 *gr.* 17633, giveth me A C, and the Secant of 10 *gr.* 101543, giveth me S C, wherefore C is the Center

Center of the Meridian, P B S, and C S his Semidiameter: ſo A F a Tangent of 20 *gr.* 36397 ſheweth F to be the Center of P D S, the twentieth Meridian from P Æ S, and A G a Tangent of 23 *gr.* 30 *m.* 43481, ſheweth G to be the Center of P ♒ S, *&c.*



The Parallels are to be drawn through the degrees, in A P, A S, and their correſpondent degrees in the general Meridian. The diſtance of the Center of each Parallel from A the Center of the Plane, is equal to the Secant of the ſame Parallel from the Pole, and the Semidiameter equal

equal to the Tangent of the ſame *degree*. As if I ſhould draw the Parallel of 80 *degrees*, which is the tenth from the Pole S, firſt I open the Compaſſes unto A C the Tangent of 10 *degrees* 17633, and this giveth me the Semidiameter of this Parallel, whoſe Center is a little from S, in ſuch diſtance as 101543 the Secants S C is longer than 10000, the Radius S A.

The Meridians and Parallels being drawn, if we number the 23 *degr.* 30 *min.* from E to ♋ Northwards, from Æ to ♎ Southward, the Line drawn from ♋ to ♎ ſhall be the Ecliptick: which being divided in ſuch ſort as the Semidiameter A P, the firſt 30 *degrees* from A to ♋ ſhall ſtand for the Sine of ♈; the 30 *degr.* next following for ♉; the reſt for ♊, ♌, ♑, *&c.* in their order.

If farther we have reſpect unto the Latitude, we may number it from E Northward unto Z, and there place the Zenith, by which and the Center, the Line drawn Z A N, ſhall repreſent the verticle Circle, and the Line M A H, croſſing it at right Angles, ſhall repreſent the Horizon, and theſe divided in the ſame ſort as A P, the Circles drawn through each *degree* of the Semidiameter A Z, Parallel to the Horizon, ſhall be the Circles of Altitude: and the Circles drawn through the Horizon and his Poles, ſhall give the Azmuths.

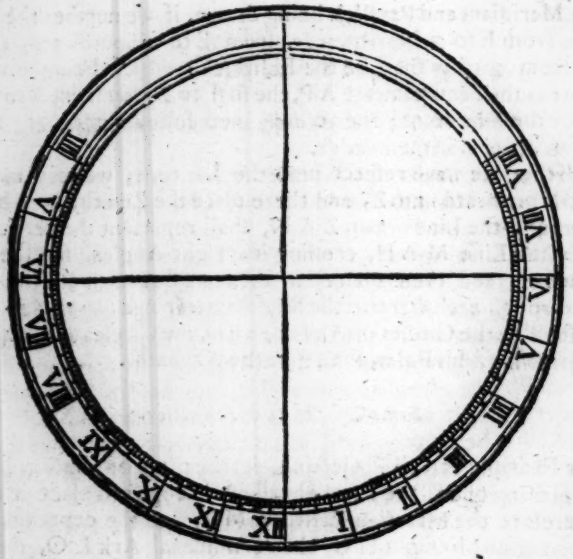
Some Uſes of this Projection.

FOr Example of this Projection, let the place of the Sun be in the beginning of ♎, the Parallel paſſing through this place is ♎ ☉ L, and therefore the Meridian Altitude M L, and the depreſſion below the Horizon at Midnight H O, the Semidiurnal Ark L ☉, the Seminocturnal Ark O ☉, the Declination A R, the Aſcenſional difference R ☉, the Amplitude of the Aſcenſion A ☉.

Or if A be put to repreſent the Pole of the World, then ſhall P Æ S E ſtand for the Equator, and P ♋ S ♎ for the Ecliptick, and the reſt which before ſtood for Meridians, may now ſerve for particular Horizons, according to their ſeveral Elevations. Then ſuppoſe the place of the Sun given to be 24 *degrees* of ♉, his Longitude ſhall be P I, his right Aſcenſion P H, his Declination H I. And if the place given be 19 *degrees* of ♌, his Longitude ſhall be P K, his right Aſcenſion P N, his Declination N K. Again, the Declination brought to the Horizon of the place, ſhall there ſhew the Aſcenſional difference, Amplitude of Aſcenſion, and the like concluſions of the Globe. But I intend

intend not here to ſhew the Uſe of the Aſtrolabe, but the Uſe of the *Sector* in Projection.

And after this manner may a Nocturnal be projected to ſhew the Hour of the Night, whereof I will ſet down a Type for the uſe of Seamen.



It conſiſts, as you ſee, of two parts, the one is a Plane divided equally according to the 24 hours of the day, and each hour into quarters or minutes, as the Plane will bear: the Line from the Center to XII, ſtands for the Meridian, and XII ſtands for the hour of 12 at midnight. The other part is a rundle for ſuch ſtars as are near the North-pole, together with the twelve months, and the days of each month fitted to the right Aſcenſion of the ſtars. Thoſe that have occaſion to ſee the South-pole, may do the like for the Southern Conſtellations, and put them in a Rundle on the back of this Plane, and ſo it may ſerve for all the World.

The

The Uſe of this Nocturnal.

The Uſe of this Nocturnal is eaſie and ready. For look up to the Pole, and ſee what Stars are near the Meridian: then place the Rundle to the like ſituation, ſo the day of the month will ſhew the hour of the Night.

SECT. III.

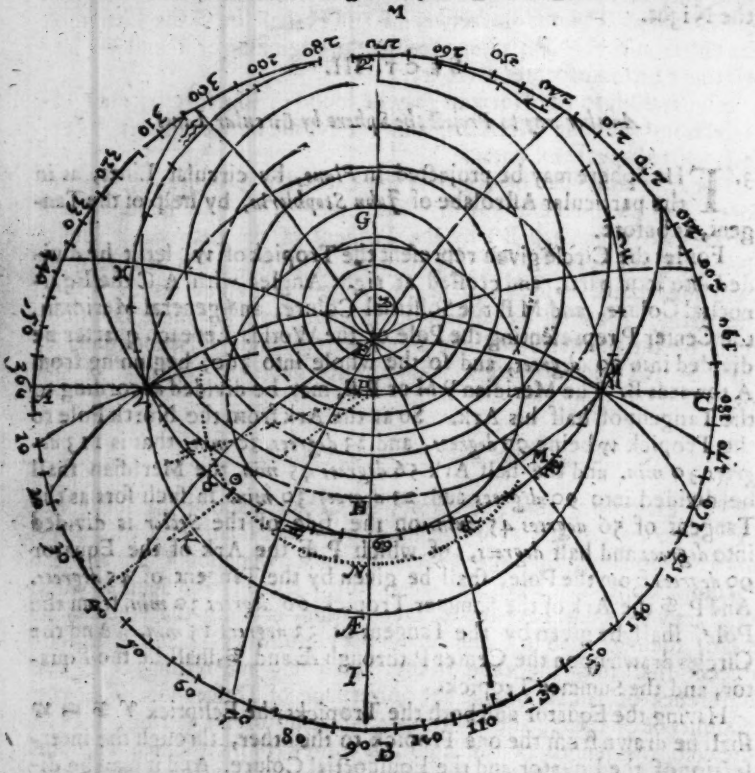
Another way to Project the Sphere by Circular Lines.

3. THe Sphere may be projected in *Plano*, by circular Lines, as in the particular Aſtrolabe of *John Stophlerin*, by help of the Tangent, as before.

For let the Circle given repreſent the Tropick of ♑, let it be divided into four parts, and croſſed at right Angles with A C the Equinoctial Colure, and M B the Solſtitial Colure, and general Meridian, the Center P repreſenting the Pole of the World. Let each quarter be divided into 90 *degrees*, and ſo the whole into 360, beginning from A towards B. The Meridian P M or P B, may be divided according to the Tangent of half his Ark. So as the Ark from the North Pole to the Tropick ♑ being 90 *degrees*, and 23 *degrees* 30 *min.* that is 113 *degrees* 30 *min.* and the half Ark 56 *degrees* 45 *min.* the Meridian ſhall be divided into 90 *degrees* and 23 *degrees* 30 *min.* in ſuch ſort as the Tangent of 56 *degrees* 45 *min.* on the ſide of the *Sector* is divided into *degrees* and half *degrees*, of which P Æ the Ark of the Equator 90 *degrees* from the Pole, ſhall be given by the Tangent of 45 *degrees*. And P ♋ the Ark of the Summer Tropick 66 *degrees* 30 *min.* from the Pole, ſhall be given by the Tangent of 33 *degrees* 15 *min.* And the Circles drawn upon the Center P through Æ and ♋, ſhall be the Equator, and the Summer Tropick.

Having the Equator and both the Tropicks, the Ecliptick ♈ ♋ ♎ ♑ ſhall be drawn from the one Tropick to the other, through the interſection of the Equator and the Equinoctial Colure. And it may be divided firſt into twelve Signs after this manner: P E the Ark of the Pole of the Ecliptick 23 *degrees* 30 *min.* from the Pole of the World, ſhall be given by the Tangent of 11 *degrees* 45 *min.* The Center of the Circle of Longitude paſſing through this Pole E ♈ and ♎, ſhall

be found at D (somewhat below B) by the Tangent of 66 *degrees* 30 *min.* Then through D draw an occult Line parallel to A C, and divide it on each side from D, in such sort as the Tangent is divided on the side of the Sector, allowing 45 *degrees* to be equall to D E, so the thirtieth *degree* from D toward the right hand, shall be the Center of the Circle of Longitude passing through E ♉ and ♎. The



sixtieth *degree*, the Center of ♊ E ♈. The thirtieth *gr.* from D towards the left hand, the Center of ♓ E ♎. The sixtieth, the Center of ♒ E ♍. And the other intermediate *degrees* shall be the Centers to divide each sign into 30 *gr.*

If

If farther we have reſpect unto the Latitude, we may (the Meridian being before divided) number it from P Northward unto H, and there place the North Interſection of the Meridian and Horizon: then the Complement of the Latitude being numbred from P Southward unto Z, ſhall there give the Zenith; and 90 *degr.* from Z Southward unto F, ſhall there give the South interſection of the Meridian and Horizon. The middle between F and H ſhall be G the Center of the Horizon ♈ H ♎ F, paſſing through the beginning of ♈ and ♎, unleſs there be ſome former errrour.

All Parallels to the Horizon may be found in like ſort by their Interſections with the Meridian, and the middle between thoſe Interſections is always the Center.

The Azimuths may be drawn as the Circles of Longitude were before. For the Circle of the firſt Verticle ♈ Z ♎, will be found at I (ſomewhat near unto B) by the Tangent of the Latitude. And if through I we draw an occult Line parallel to A C, and divide it on each ſide from I, in ſuch ſort as the Tangent is divided on the ſide of the *Sector*, allowing 45 *degrees* to be equal to I Z, theſe Diviſions ſhall be the Centers, and the diſtance from theſe Diviſions unto Z, ſhall be the Semidiameters whereon to deſcribe the reſt of the Azimuths.

Some Uſes of this Projection.

FOr example of this Projection, let ☉ the place of the Sun given be 10 *degr.* of ♉: a right Line drawn from P through this place unto the Equator, ſhall there ſhew his right Aſcenſion ♈ K, and his Declination K ☉. Then may we on the Center P and Semidiameter ☉ P draw an occult Parallel of Declination, croſſing the Horizon in L, M. the Meridian in G and N. So the right Lines P L and P M produced, ſhall ſhew the time of the Suns riſing and ſetting, ♈ Q the difference of Aſcenſion, R the difference of Deſcenſion, ♈ L the Amplitude of riſing, and ♎ M the Amplitude of his ſetting, L N M ſheweth the length of the night, Z G ſheweth his diſtance from the Zenith at noon, H N his depreſſion below the Horizon at midnight. And then having the Altitude of the Sun at any time of the day, the Interſection of the parallel of Altitude with the parallel of Declination, ſheweth the Azimuth, and a right Line drawn from P through this Interſection, giveth the hour of the day.

Sect. IV.

A third way to Project the Sphere in Plano, by Circular Lines.

4. THe Sphere may be Projected in *Plano* by Circular Lines, after the manner of the old concave Hemiſphere, by the help of the Tangent on the ſide of the *Sector*.

For let the Circle given repreſent the Plane of the Horizon, let it be divided into four parts, and croſſed at right Angles with S N the Meridian, and E V the Verticle; ſo as S may ſtand for the South, *N.* for the North, E for the Eaſt, W the Weſt part of the Horizon, and the Center Z repreſent alſo the Zenith. Let each quarter of the Horizon be divided into 90 *degrees*, and ſo the whole into 360 *degrees*, beginning from N, and ſetting to the numbers of 10,20,30,*&c.* 90 at E, 180 at S, 270 at W, 360 at N.

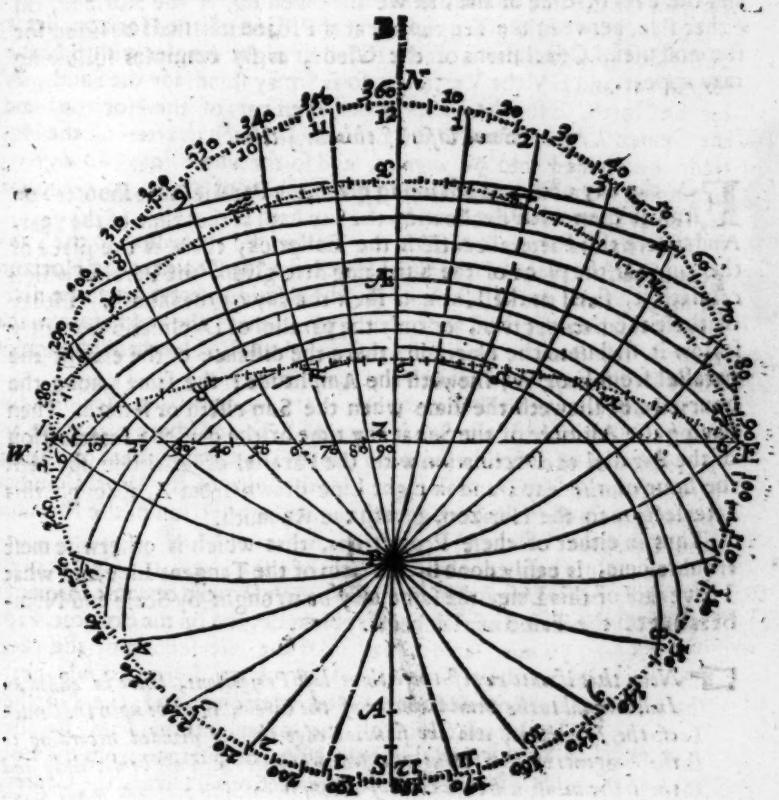
The Semidiameters Z N, Z S, may be divided according to the Tangent of half their Arks: ſo as the Ark from the Zenith to the Horizon being 90 *gr.* and the half Ark 45 *gr.* the Semidiameters are to be divideed in ſuch ſort as the Tangent of 45 *gr.* as was ſhewed before in the ſecond Projection. And if from Z we draw Circles through each of theſe Diviſions, they ſhall be Parallels of Altitude.

Then having reſpect unto the Altitude, we may (the Meridian being before divided) number it from Z to Æ, and there place the Interſection of the Meridian and Equator. The Complement of the Latitude from Z unto P, ſhall there give the Pole of the World, and 90 further from P, ſhall there give the other interſection of the Meridian and Equator.

The middle between theſe interſections ſhall be A the Center of the Equator, paſſing through E and W, unleſs there be ſome former errour. The interſections of the Tropicks depend on the Equator. From Æ 23 *degrees* 30 *min.* farther ſhall be ♑ the interſection of the Meridian and the Southern Tropick. From Æ 23 *degrees* 30 *min.* nearer ſhall be ♋, the Interſection of the Meridian and the Northern Tropick. The Interſections of the other intermediate Parallels, ſhall be given in like ſort, by their *degrees* of diſtance from the Equator, and the middle between thoſe Interſections is always the Center.

The Hour Circles may be here drawn as the Azimuths in the third Projection. For the Center of E P W, the hour of 6, will be found

at B, (somewhat near unto N) by the Tangent of the Latitude. And if through B we draw an occult Line parallel unto E W, and divide it on each side from B, in such sort as the Tangent is divided on the side of the *Sector*, allowing 45 *degrees* to be equal to B P, and 15 *degrees*, for every hour, those Divisions shall be the Centers, and the distance from the Divisions unto P, shall be the Semidiameters, whereon to describe the rest of the hour Circles.



The

The Ecliptick may be drawn as the Equator. For the Center of that half which hath Southern Declination, ſhall be given by the Tangent of the Altitude, which the Sun hath in his entrance into ♈. And the Center of the other half by the Tangent of his Altitude, at his entrance into ♎, and it may be divided, as in the former Projection, or elſe by Tables calculated to that purpoſe.

To theſe Circles thus drawn, if we ſhall add the months of the year, and the days of each month, as we may well do, at the Horizon, on either ſide, between the Tropicks: this Projection ſhall be fitted for the moſt uſeful Concluſions of the Globe, as by examples following may appear.

Some Uſes of this Projection.

FOr the day of the month being given, the Parallel that ſhooteth on it, doth ſhew what declination the Sun hath at that time of the year. And where this Parallel croſſeth the Ecliptick, there is the place of the Sun. Or the place of the Sun being firſt given, the Parallel which croſſeth it, ſhall at the Horizon ſhew the day of the month. Either of theſe then being given, or only the parallel of Declination, we may follow it, firſt unto the Horizon, there the diſtance of the end of the parallel from E or W, ſheweth the Amplitude; the ſame among the hour-circles ſheweth the time when the Sun riſeth or ſetteth. Then having the Altitude of the Sun at any time of the day, the Interſection of the Parallel of Declination with the Parallel of Altitude ſheweth the hour of the day; and a right Line drawn from Z, through this Interſection to the Horizon, giveth the Azimuth.

Thus in either of theſe Projections, that which is otherwiſe moſt troubleſome, is eaſily done by the help of the Tangent Line, and what I have ſaid of this Line, the ſame may be wrought by Scale and Numbers out of the Table of Tangents.

☞ *Note, that if unto any of theſe three laſt Projections, there be added an Index equal to the Semidiameter of the Circle, to move upon the Center of the Projection, and the fiducial edge thereof divided according to the Tangents of half Arks, the Semidiameters need not be divided, and the Inſtruments will then be fitly accommodated to perform many Concluſions of the Sphere.* ☜

☞By the former ways of Projecting of the Sphere, the whole Art of Dialling may be performed upon any of them, but especially upon this last, which may be fitted to the Horizon of any place, the manner whereof in this place I shall briefly deliver.

1. *For an Horizontal Dial.*

If streight Lines be drawn from the Center of the Projection through the intersections of the hour Circles with the outermost Circle or Horizon, those Lines so drawn shall be the true hour Lines of an Horizontal Dial in that Latitude for which the Projection was made, for the hour Circles cut the Horizon at those degrees of distance.

2. *For an Erect direct North or South Dial.*

If an Index be divided as the Semidiameter of the Projection Z W is, on both sides, and laid upon the same Diameter W E, the hour Lines of the Projection will cut the same Index in such degrees from the Meridian on either side, as the hour Lines on such a North or South Dial ought to have upon the Plane: As,

				deg.	min.	
The hour Lines of	11	1	do cut the Index at	9	28	From the Meridian.
	10	2		19	54	
	9 and	3		31	54	
	8	4		47	09	
	7	5		66	42	

And these are the true hour distances for a North or South Plane in this Latitude of 51 *degrees* 30 *min.* for which this Projection was made.

3. *For a Vertical Declining Dial.*

Suppose an upright Plane to decline from the South Westward 24 *degr.* 20 *min.* Such a Plane is described in the third Book of this Treatise *Chap.* 7. If you lay your Index to 24 *degr.* 20 *min.* counted from E towards S, and there keep it fixed, the hour Lines of the Projection

Projection will cut the Index in theſe degrees from the Meridian, on either ſide thereof, at which they are to be drawn upon the Dial Plane.

4. *For direct Incliners.*

Let the Inclining Plane be projected upon the Scheme, a Ruler laid to the Pole of the inclining Plane, and to the ſeveral Points where the hours croſs the Plane, the Ruler will cut the outermoſt Circle in the degrees that the hour Lines ought to have upon ſuch an inclining Plane.

Thus let the Circle W Æ E, which is the Equinoctial Circle, repreſent a Plane inclining to the Horizon, a Ruler laid to the Pole of the World (which is the Pole of the Equinoctial Circle) and the ſeveral interſections of the hour Circles with this Circle, ſhall cut the outermoſt Circle in every fifteenth degree, and ſuch diſtance ought each hour have from other upon the Plane.

5. *For Declining inclining Planes.*

Let a Plane decline from the South Weſtward 24 *degrees* 20 *min.* and incline to the Horizon Northward 36 *degrees*, ſuch a Plane is repeſented in the Diagram of the ſeventh Chapter of the third Book of this Treatiſe, by the Circle B M D. Now a Ruler laid upon the Pole of this Plane, (which is in the Line Q H, 90 *degrees* diſtant from M) and the interſections of the hour Circles with the Plane, ſhall cut the primitive or horizontal Circle in the degrees of diſtance that the reſpective hour Lines of ſuch a declining inclining Plane ought to have upon the Dial Plane. ☞

☞ SECT. 5.

Of Projecting of the Sphere upon Oblique Circles.

IN the four firſt Sections of this Chapter, Mr *Gunter* hath ſhewed how to Project the Sphere in *Plano* upon the principal Great Circles of the Sphere, *viz.* Twice upon the Plane of the Meridian, once upon the Tropick of ♑, or the Equinoctial, (parallel thereto) and laſtly upon the Horizon.

To

To theſe Projections I think it will not be impertinent (but very beneficial and ſatisfactory to the Reader) to ſhew how the Sphere may be Projected in *Plano* upon any Oblique Circle, as upon any Plane whatſoever and howſoever ſituate, for all or moſt Diall Planes are Obdique Circles, and are Horizons in ſome part of the World or other. As for Example, A Dial Plane declining from the South Weſtward 24 *degrees* 20 *min.* and inclining Northward 36 *degrees* (ſuch is the Diall Plane in the tenth Chapter of the third Book of this Treatiſe of Dialling) will in ſome place or other be an Horizontal Plane: And by projecting of the Circles of the Sphere in their true poſitions upon this Oblique Plane, you ſhall not only diſcover in what Longitude and Latitude this will be an Horizon, but will alſo delineate out unto you the places of the Hour-lines proper for this declining inclining Plane, in a quite different manner and form than that which Mr *Gunter* hath ſhewed how to make the Diall in the forementioned tenth Chapter of the third Book, by drawing the Plane upon the Horizontal Projection for this Latitude. And ſeeing the difference of the two ways of working are ſo various, and the variety that will appear in the placing of the Circles of the Sphere in their true poſitions upon ſuch an Oblique Plane cannot but be both beneficial and delightful, I ſhall here inſert the manner how the ſame may be effected, not only upon this, but upon any other Oblique Plane whatſoever.

To proceed then, Let the Circle H X O D, repreſent a Dial Plane declining from the South Weſtward 24 *degrees* 20 *min.* and inclining Northward 36 *degrees.*

1. Draw the Diameter H O, and croſs it at right Angles with the Line C F meeting in the Center Q.

2. Take the half Tangent of 36 *degrees*, the Planes inclination, and ſet it from Q to Z, ſo ſhall Z be the Zenith of the Place.

3. Take the half Tangent of 54 *degrees*, the Complement of the Planes inclination, and ſet that from Q to B, ſo ſhall B be the Point through which the Horizon of the place muſt paſs.

4. Take the Tangent of 36 *degrees* the Plains Inclination, and ſet it from Q to C. Or take the Secant of 36 *degrees*, and ſet it from B to C, ſo ſhall C be the Center of the Horizon H B O.

5. Take the Tangent of 54 *degrees*, the Complement of the Inclination, and ſet it from Q to F.

6. Take 24 *degrees* 20 *min.* out of your Line of Chords and ſet that diſtance from H to c, from D to d, and from O to c.

7. A Ruler laid from Z to c, d, and e, will give the Points W, S and E, for the West South and East Points thereof.

8. Draw F G perpendicular to Q F, or parallel to H O, and extend it as far as you shall see requisite.

9. Draw a Line through the Points E and W extending it till it cross the Line F G last drawn, at G, so shall G be the Center of the Meridian of the place represented by P Z S.

10. Lay a Ruler from W to Z and it will cut the Circle in *a*, from which Point *a* set 38 *degrees* 30 *min.* the Co-latitude to *b*, and a Ruler laid from W to *b* will give the Point P in the Meridian for the Pole of the World.

11. Set 90 *degrees* of your Chords from *b* to *f*, and from *f* to *g*. A Ruler laid from W to *f* gives Æ in the Meridian Circle, for the Equinoctial Point, and from W to *g* gives M for the South Pole, and a right Line drawn through P Q and M shall be the Axis of the World.

12. Through the Points W Æ E draw the Equinoctial Circle, to find the Center whereof,

13. Divide W E into two equal parts in R, and raise the Perpendicular R T, drawing it forth till it meet with Q P being extended, here represented by the two Lines R T and Q V, whose meeting shall be the Center of the Equinoctial, which Q T extended would be equal to the Secant of the height of the Pole above the Plane. Or if from T you draw a Line through C it will intersect Q V in the Center of the Equinoctial also.

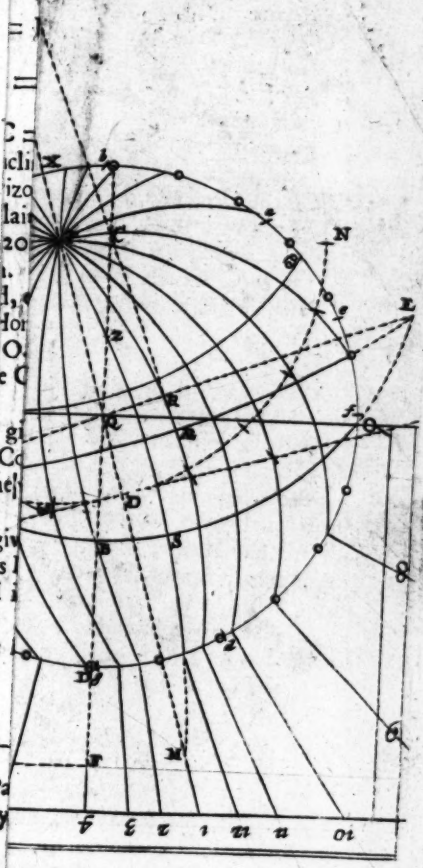
14. Divide M P in two equall parts in D, and draw D G at right Angles to P M, and extend D G infinitely.

15. Upon P, at the distance P D (or any other) describe the Semicircle L D N, and laying a Ruler from P to G the Center of the Meridian, it will cut the Semicircle L D N in L, at which Point L begin to divide L D N into twelve equal parts, and a Ruler laid from P through each of those equal parts shall give the Tangents of 15, 30, 45, *&c.* upon the Tangent G D and shall be the Centers of the several Meridians, G being the Center of twelve a clock, or the Meridian of the Place.

15. From the Center Q through the Points where the several Meridians do cut the Primitive Circle draw right Lines, and those Lines shall be the true hour Lines of a South Plane declining Westward 24 *degrees* 20 *min.* and inclining Northward 38 *degrees* in the Latitude of 51 *degrees* 30 *min.*

And

	d	′
South Inclin Nor	36	00
Declin West	24	20
Latitude	51	30



A brief Synopsis *of this Oblique* Projection.

H X O D, the declining inclining Plain.

Q Z = ½ Tang. 36 *d.* = Plains inclination.

Q B = ½ Tang. 54 *d.* = Co-Plains inclination.

Q C = Tang. 36 *d.* Or C = Secant of 36 *d.* = Plains inclin. and is the Center of the Horizon.

Q F = Tan. 54 *d.* = Co-Plains inc.

H c = D d = O e = 24. 20 *m.* = to the Plains Declination.

A Ruler laid from Z to d, e, will give W, S and E in the Horizon:

G F ⊥ to Q F or ‖ to H O.

E W extended, gives G the Center of the Meridian.

A Ruler laid from W to Z gives a.

a b = Chord 38 *d.* 30 *m.* = Co-Lat.

W b gives P for the Pole of the world

b f = f g = Chord 90 *d.*

A Ruler laid from W to f gives Æ the Equinoct. W g, gives M the South Pole, and P Q M is the Axis of the World.

Through W Æ E, draw the Equinoctial Circle.

W R = R E = ½ Tang. W E.

R T ⊥ to W E. Or, draw R T through the Center of the Horizon, and its intersection with Q V will be the Center of the Equinoctial.

Q V and R T extended, and intersecting, gives the Center of the Equinoctial = Sec. of the Poles height above the Plain.

Divide M P in 2 = parts in D and draw D G infinitely.

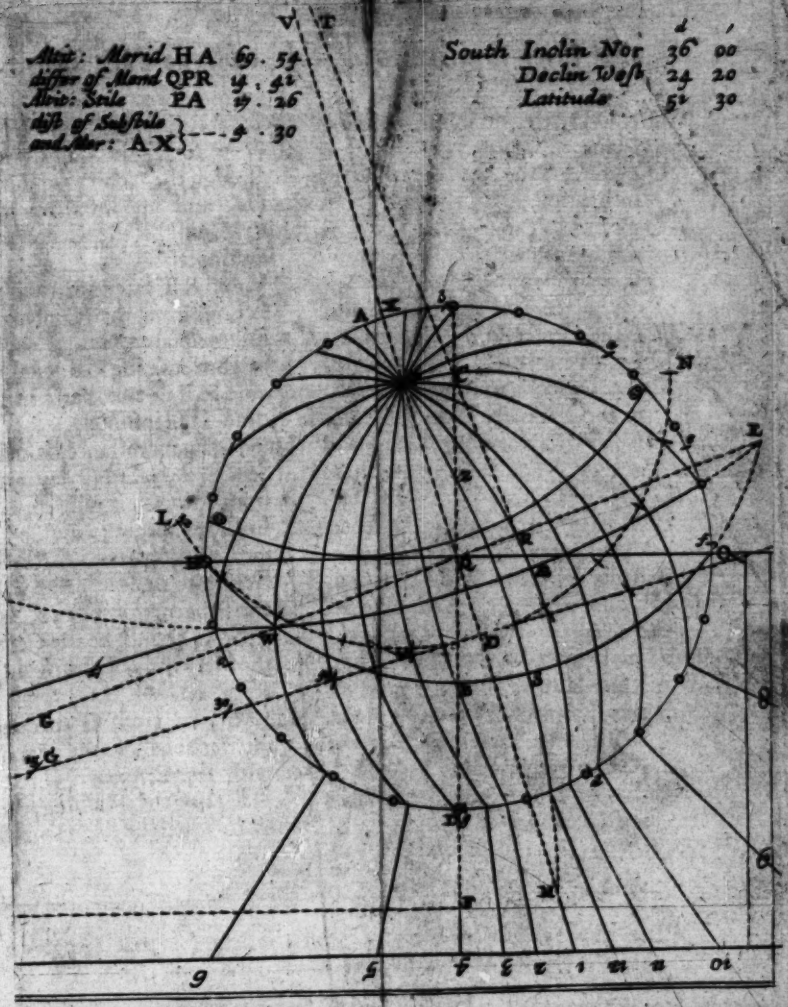
Upon P, at any distance, describe the Semicircle L D N, lay a Ruler from P to G, it will cut L D N in L, at L begin to divide L D N into 12 = parts. A Ruler laid from P to those = parts, will give the Tangents of 15, 30, 45, *&c.* upon G D, and be the Centers of the Meridians, G being the Center of 12.

Lines drawn from Q through the intersections of the Meridians with the Primitive Circle (representing the declining inclining Plain) shall be the Hour-Lines.

Place this *Synopsis* against Page 74 of the Sector, so that it, together with the Scheme, may lie out when the Book is shut.

Altit: Merid	HA	69	54
differ of Mand	QPR	4	42
Altit: Stile	PA	19	26
dift of Subftile and Alt:	AX	9	30

		d	′
South Inclin Nor		36	00
Declin Weſt		24	20
Latitude		51	30

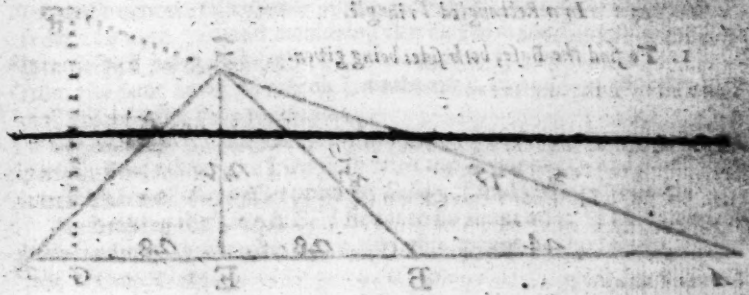


And

And from this Scheme may further be found that,

		degr	min.
1.	The Elevation of the Meridian H A is	69	54
2.	The difference of Meridians Q P R is	14	41
3.	The height of the Stile P A is	17	26
4.	The distance of the Substile and Meridian A X is	4	39

Note, In like manner might be inserted in this Projection, the Tropicks and other Parallels of the Suns place or Declination; The Azimuths, Almicanthar's, the Ecliptick, and other (either small or great) Circles; as is instanced in the Scheme by the Tropick of Cancer, which is thereon described; but of this Oblique Projection I have said enough in this place.



CHAP. IV.

Of the Resolution of right-lined Triangles.

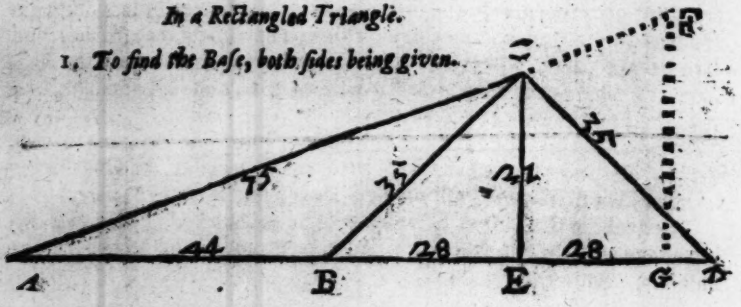
IN all Triangles there being six parts, *viz.* three Angles, and three sides, any three of them being given, the rest may be found by the *Sector*.

As may appear by the *Prop.* following, wherein for our practise we may use these Triangles C E A, C E B, C E D, all which are Rectangled in E. And A G F Rectangled in G. All the rest consist of Oblique Angles.

Ang.	*Gr.*	*M.*	*S.*	*Line.*	*Parts.*
E	90	0	0	*A C*	75
G	90	0	0	*A F*	100
A	16	15	36	*F G*	28
D	36	52	12	*C E*	21
B	36	52	12	*C D*	35
B	143	7	48	*B B*	35
A F G	73	44	12	*A G*	96
A C E	73	44	12	*A E*	72
A C B	20	36	36	*A B*	44
B C E	53	7	48	*B D*	28
E C D	53	7	48	*A D*	28
B C D	106	15	36	*B E*	56
A C D	126	52	12	*E D*	100

In a Rectangled Triangle.

1. *To find the Base, both sides being given.*



Let the *Sector* be opened in the Line of *Lines* to a right Angle (as before was shewed *Cap.* 2. *Prop.* 7.) then take out the sides of the Triangle, and lay them, one on one *Line*, the other on the other *Line*, so as they meet in the Center, and mark how far they extend. For the *Line* taken

taken from the terms of their extenſion, ſhall be the Baſe required, *viz.* the ſide oppoſite to the right Angle.

Or add the ſquares of the two ſides (as in *Prop.* 4. Superfic.) and the ſide of the compound Square ſhall be the * Baſe.

* *Note that I call the longeſt ſide of the Triangle the Baſe.*

As if the Lines A E, C E, ſhould be the ſides about the right Angle, and it were required to find the Baſe ſubtending the right Angle.

Firſt, I ſet the Line of *Lines* to a right Angle by applying the whole Line of 10 from 6 in the one Line, to 8 in the other. Then if the greater of the two Lines given be leſs than the Line of *Lines*, I take the greater of them A E, and tranſfer it with the Compaſſes into one of the Lines of *Lines*, and find, that, in my *Sector* (which is 14 Inches long, and ſo, the Line of *Lines*, almoſt 7 Inches) it reacheth from the Center to 518.

Again, I take the leſſer Line C E, and tranſfer it into the other Line of *Lines*, and find, that it reacheth from the Center unto 151, wherefore I take the diſtance from 151 unto 518, and ſuch is the length of the Baſe A C required.

If either of the Lines given be too large for the *Sector*, then I may meaſure them by Feet or Inches, as ſuppoſe I find the length of A E to be about 720, and of C E 210. Then in the Line of *Lines* (being ſet, one Perpendicular to the other, as before) I extend the Compaſſes from 210 unto 710; and meaſuring this extent in the Line of *Lines*, find it to be 750 parts, wherefore I prick down 750 parts in the Line A C, from the ſame Scale by which I meaſured A E, and C E. So, this Line A C ſhall be the Baſe required.

In working by the Line of *Superficies*, I need no opening of the *Sector*. For, taking the Line C E with my Compaſſes, and meaſuring it in the Line of *Superficies* upon my *Sector*, I find it near 13 parts.

Then taking the Line A E, I find it to be about 269. Theſe two being added together make 292: and this extent is the length of the Baſe A C required.

2. *To find the Baſe, by having the Angles, and one of the ſides given.*

Take the ſide given, and turn it into the parallel Sine of his oppoſite Angle; ſo the parallel Radius ſhall be the Baſe.

As if the Line A E were the ſide of a Rectangle Triangle oppoſite

oppoſite to an Angle of 73 *gr.* 45, and it were required to find the Baſe.

Firſt, I take the ſide A E with my Compaſſes, and ſet it over in the Sines of 73 *gr.* 45. So, the Parallel Radius taken from between 90 and 90, will give the Baſe A C required.

If the ſide given be ſuch as cannot well be fitted over in the Sines of his oppoſite Angle, I may meaſure it by feet or inches, and ſuppoſe I find the length of A E to be 720, then would I take 720 parts out of the Line of Lines, and make it a Parallel Sine of 73 *gr.* 45. So, the Parallel Radius taken from between 90 and 90, and meaſured in the Line of *Lines* will be found to be about 750 parts: Wherefore, I prick down 750 in the Line A C, by the ſame Scale, whereby I meaſured A E: and this Line A C ſhall be the Baſe required.

3. To find a ſide by having the Baſe and the other ſide given.

Let the Sector be opened in the Lines of *Lines* to a right Angle, and the ſide given laid on one of thoſe Lines from the Center: then take the Baſe with a pair of Compaſſes, and ſetting one foot in the term of the given ſide, turn the other to the other Line of the Sector, and it ſhall there ſhew the ſide required.

Or take the Square of the ſide out of the Square of the baſe (as in *Prop.* 4. *Superfic.*) and the ſide of the remaining Square ſhall be the ſide required.

Thus having A C for the Baſe, and C E, for the ſide of a Rectangle Triangle, the other ſide will be found to be A E.

Or, if A C, being meaſured, be 750, and C E, 270, the other ſide A E will be found to be 720.

To find a ſide having the Baſe, and the Angles given.

Take the Baſe given, and make it a Parallel Radius, ſo the parallel Sines of the Angles, ſhall be the oppoſite ſides required.

Thus in the Rectangle A E C, if A C be made a Parallel Radius, the Parallel Sine of 73 *gr.* 45, will give the ſide A E; and the Parallel Sine of 16 *gr.* 15. will give the other ſide C E.

5. To

5. *To find a ſide by having the other ſide and the Angles given.*

Take the ſide given, and turn it into his Parallel Sine of his oppoſite Angle: ſo the Parallel Sine of the Complement ſhall be the ſide required.

Thus in the Rectangle D E C, if C E be made a Parallel Sine of 53 *gr.* 8 *m.* the parallel Sine of 36 *gr.* 52 *m.* will give the ſide E D. and the Parallel Sine of 90 *gr.* will give the Baſe C D.

6. *To find the Angles by having the Baſe and one of the ſides given.*

Firſt, take out the Baſe given, and laying it on both ſides of the Sector, ſo as they may meet in the Center, and mark how far it extendeth. Then take out the lateral Radius, and to it open the Sector in terms of the Baſe. This done, take out the ſide given, and place it alſo on the ſame Lines of the Sector from the Center. For the Parallel taken in the terms of this ſide, ſhall be the Sine of his oppoſite Angle.

Or take the baſe given, and make it a Parallel Radius; then take the ſide given, and carry it parallel to the Baſe, till it ſtay in like Sines: ſo they ſhall give the quantity of the oppoſite Angle.

Thus in the Rectangle A E C, having the Baſe A C, and the ſide A E, you may find the Angle C A E, to be 16 *gr.* 15 *m.*

7. *To find the Angles by having both the ſides given.*

Take out the greater ſide, and lay it on both ſides of the Sector, ſo as they meet in the Center, and mark how far it extendeth. Then take the other ſide, and to it open the Sector in the terms of the greater ſide; ſo the Parallel Radius ſhall be the Tangent of the leſſer Angle. The third Angle is always known by the Complement.

Thus in the Rectangle D E C, having the ſides C E, and E D, you may find the leſſer Angle E C D to be 36 *gr.* 52 *m.* and therefore the other Angle E D C to be 53 *d.* 8 *m.*

8. *The Radius being given, to find the Tangent and Secant of any Ark.*

9. *The*

9. *The Tangent of any Ark being given, to find the Secant thereof, and the Radius.*

10. *The Secant of any Ark being given, to find the Tangent thereof, and the Radius.*

The Tangent, and the Secant, together with the Radius of every Ark, do make a right Angle Triangle; whose sides are the Radius and Tangent, and the Base always the Secant; and the Angles always known by reason of the given Arks. As in the Rectangle A E C, if on the Center A, and Semidiameter A E, you describe a Circle, then make A E, to be the Radius, and E C, a Tangent of 16. 15 and A C a Secant of 16 *gr.* 15 *m.*

If you describe a Circle on the Center C, and Semidiameter C E, then is C E the Radius and E A, a Tangent of 73 *gr.* 45 *m.* and C A a Secant of 73. 45.

Wherefore the Solution is the same with those before.

In any right-lined Triangle whatsoever,

11. *To find a side by knowing the other two sides, and the Angle contained by them.*

Let the *Sector* be opened in the Lines of *Lines* to the Angle given as I shewed before, *Cap.* 2. *Prop.* 7. Then take out the sides of the Triangle, and laying them the one on the one Line, the other on the other, so as they meet in the Center, mark how far they extend. For the Line taken between the terms of their Extension, shall be the third side required.

As if A C and A D were two sides of a right lined Triangle containing an Angle of 16 *gr.* 16 *m.* and it were required, to find the third side subtending this Angle.

First, I set the Lines to an Angle of 16. 16 *m.* by applying the Sine of 8 *gr.* 8 *m.* over in the Points of 50 and 50, in the Line of Lines. That done, I take the longer Line A D, and transfer it with my Compasses into one of the Lines of *Lines*, and find it to reach from the Center to 720.

Again, I take the lesser Line A C, and transfer it into the other Line of *Lines*, where it reacheth from the Center to 540, wherefore I take the distance from 540 to 720, and such is the length of the third side C D required.

Or

Or (if the Lines be given in meaſure) A D 100, and A C 75. I extend the Compaſſes from 100 to 75, and meaſuring this extent in the Line of Lines, find to be 35. Whereupon I take 35 parts out of the Scale, by which A C, and A D were meaſured, and prick them down in the Line C D. So, this Line C D ſhall be the third ſide required.

12. *To find a ſide by having the other two ſides, and one of the adjacent Angles, ſo it be known which of the other Angles is Acute or Oblique.*

Let the *Sector* be opened in the Line of *Lines* to the Angle given, and the adjacent ſide laid on one of thoſe Lines from the Center; then take the other ſide with a pair of Compaſſes, and ſetting one foot in the term of the former given ſide, turn the other to the other Line of the *Sector* which here repreſenteth the ſide required, and it ſhall croſs it in two places; but with which of them is the term of the ſide required, muſt be judged by the Angle.

As if in the Triangle following, the ſide A C being given, and the ſide C D and the Angle C A D 16 *gr.* 16 *m.* it were required to find the ſide A D.

Firſt, I open the Sector in the Line of *Lines* to an Angle of 16 *gr.* 16 *m.* and laying the adjacent ſide from the Center A, find where it extendeth in C. Then I take the other ſide C D with the Compaſſes, and ſetting one foot in C, and turning the other to the other Line of the *Sector*, I find that it doth croſs it both in B and D.

Or, (if the Lines be given in meaſure) A C 75, and C D 35; I may take 35 out of the Line of *Lines*, and ſetting one foot in 75, I ſhall find the other foot to croſs the other Line of the *Sector*, both at 44 (anſwerable to A B) and at 100 (anſwerable to A D.)

So that it is uncertain whether the ſide required be A B or A D, only it may be judged by the Angle. For if the inward Angle where they croſs be Obtuſe, the ſide required is the leſſer; if it be Acute, it is the greater.

13. *To find a ſide by having the Angles, and one of the other ſides given.*

Take the ſide given, and turn it into the Parallel Sine of his oppoſite Angle; ſo the Parallel Sines of the other Angle ſhall be the oppoſite ſides required.

As if in the Triangle A B C, having the ſide A D, and knowing the Angle C A B to be 16 *gr.* 16 *m.* and the Angle A B C to be 143 *deg.* 8 *m.* it were required to find the other two ſides, A C, and B C.

The three Angles of a right-lined Triangle, are always equal to 180 *gr.* wherefore I add 16 *gr.* 16 *m.* unto 143 *gr.* 8 *m.* and by the remainder to 180 *gr.* find the third Angle A C B oppoſite to the known ſide A B, to be 20 *gr.* 36 *m.* Then I take the ſide A B, and make it a Parallel Sine of 20 *gr.* 36 *m.*

So, his Parallel Sine of 16 *degr.* 16 *m.* will be the the ſide B C; and the Parallel Sine of 143 *degr.* 8 *m.* will be the ſide A C.

Or if meaſuring the ſide A B, I find it to be 44; I may take 44 parts, either out of the Line of Lines, or o t of any other Scale of equal parts, and make it a Parallel Sine of 20 *gr.* 36 *m.* So his Parallel Sine of 16 *gr.* 16 *m.* meaſured in the ſame Scale, will give 35 for the length of the ſide B C: and the Parallel Sine of 36 *gr.* 52 *m.* will give 75, for the length of the other ſide A C.

When the Angle comes to be above 90 *gr.* the Sine of 80 *gr.* doth ſtand for a Sine of 100 *gr.* and the Sine of 70 *gr.* for a Sine of 110 *gr.* and ſo the reſt; for thoſe, which are their Complements to 180 *gr.*

14. *To find the proportion of the ſides by having the three Angles.*

Take the lateral Sines of the Angles, and meaſure them in the Line of Lines. For the numbers belonging to thoſe Lines do give the proportion of the ſides.

Thus, in the two equi-angle Triangles A E C, A G F, if you take the lateral Sine of 90 *gr.* for the right Angle at E and G, and meaſure it in the Line of *Lines*, you ſhall find it to be 100. Then take the lateral Sine of 16 *gr.* 16 *m.* for the common Angle at A, you ſhall find it to be 28. Take the lateral Sine of 73 *gr.* 44 *m.* For the third Angle at C and F, you ſhall find it to be 96. Such therefore is the proportion of the ſides.

As 100: 96. 28:: So are 75: 72. 21.

15. *To find an Angle, by knowing the Three ſides.*

Let the two containing ſides be laid on the Lines of the *Sector*, from the Center, one on one Line, and the other on the other; and let the third ſide, which is oppoſite to the Angle required, be fitted over in their terms, ſo ſhall the *Sector* be opened in thoſe Lines to the quantity of the Angle required.

The

The quantity of this Angle is found as in *Cap.*2. *Prop* 8. Thus having the three ſides of the Triangle A C D, to find the Angle at A, I take the two containing ſides A D, A C, and transfer them with my Compaſſes into the Line of *Lines*: where I find the one to reach from the Center, to 72; the other, to 54.

Then I take C D, (the ſide oppoſite to the Angle at A) and ſet that over between 72 and 54.

Or if the three ſides be given in meaſure A D 103; A C 75; C D 35: I might take 35 for the ſide C D out of the Line of *Lines*, and ſet that over from 100 to 75. This done I take the diſtance between 50 and 50 and meaſuring it in the Line of *Sines* I find it to be about 8 *gr.* 8 *m.* the double whereof is 16 *gr.* 16 the Angle required.

16. *To find an Angle, by having two ſides, and one adjacent Angle.*

Firſt take out the ſide oppoſite to the Angle given, and laying it on both ſides of the Sector, ſo as they meet in the Center, mark how far it extendeth; then take out the lateral Sine of the Angle, and to it open the Sector in the terms of the firſt ſide: this done take out the other ſide given, and place it alſo on the ſame Lines of the Sector from the Center, for the Parallels taken in the terms of this ſide; ſhall be the Sine of the Angle oppoſite to the ſecond ſide.

Or take out the ſide oppoſite to the Angle given, and make it a Parallel Sine of that Angle: then take the other ſide given and carry it Parallel to the former, till it ſtay in like Sines: ſo they ſhall give the quantity of the Angle oppoſite to the ſecond ſide.

Thus in the Triangle A C D, knowing two ſides A C, C D, with the Angle C A D oppoſite to the ſide C D, you may find the Angle A D C oppoſite to the other known ſide A C, to be about 36 *gr.* 52 *m.*

17. *To find an Angle by having two ſides, and the Angle contained by them.*

Firſt find the third ſide by the 11 *Prop.* and then the Angles may be found by the 15 or 16 *Prop.*

For obſervation of Angles, the *Sector* may have ſights ſet on the movable foot: ſo that by looking through them, the edges of the Sector may be applied to the ſides of the Angle.

For measuring of the side of lesser Triangles, any Scale may suffice, either of feet, or inches, or lesser parts. But for greater Triangles, especially for plotting of grounds. I hold it fit to use a chain of four Perches in length, each Perch divided into 25, and the whole Chain an hundred Links, wherein, if the whole Chain be (according to 16½ foot in a Perch) 66 foot, (that is 792 inches) each several Link will be seven inches and $\frac{22}{100}$.

If (according to 18 in the Perch) the whole Chain be 72 feet in length (that is, 864 inches) then each several Link will be eight inches and $\frac{64}{100}$.

For so the length being multiplied into the breadth, the five last Figures give the content in Roods and Perches by this Table; the other Figures towards the left hand do shew the number of Acres directly.

As in a long Square, where the length is 24 Chains ½ the breadth 13 Chains ½ the usual way is, to resolve the Chains into Perches: So the length is 97 Perches, and the breadth 54 Perches. These multiplied one into the other make 5238 square Perches, and those (divided by 160) give 32 Acres, 2 Roods, and 38 Perches for the content required.

But, reckoning by Chains and Links, the length is 24 Chains 25 Links, the breadth 13 Chains 50 Links. These multiplied one into the other make 32. 73750 square Links. Then, cutting off the five last Figures, I find 32 Acres 73750 *lin.* such as an 100000 do make an Acre, Of which 70000 are equal to two Roods 32 Perches: and the rest 3750 equal to 6 Perches more (as appeareth by this Table.) So, the whole content is 32 Acres, 2 Roods, 38 Perches, as before.

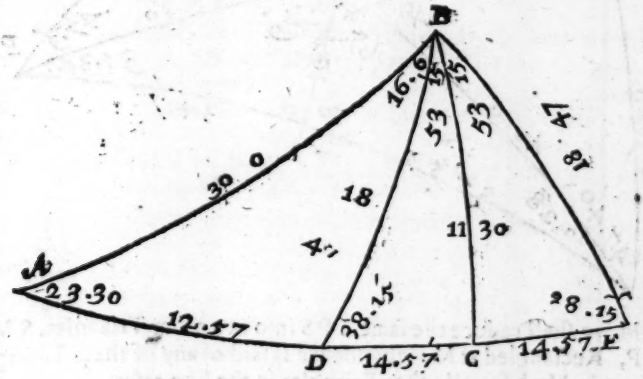
Link.	*R.*	*P.*
100000	4	0
90000	3	24
80000	3	8
70000	2	32
60000	2	16
50000	2	0
40000	1	24
30000	1	8
20000	0	32
10000	0	16
9375	0	15
8750	0	14
8125	0	13
7500	0	12
6875	0	11
6250	0	10
5625	0	9
5000	0	8
4375	0	7
3750	0	6
3125	0	5
2500		4
1875	0	3
1250	0	2
624	0	1

CHAP.

CHAP. V.

Of the Resolution of Spherical Triangles.

FOr our practise in Spherical Triangles, let A be the Equinoctial Point, A B an Ark of the Ecliptick, representing the Longitude of the Sun in the beginning of ♉, B C an Ark of the Suns Declination from the Equator, and A C an Ark of the Equator representing the right Ascension.

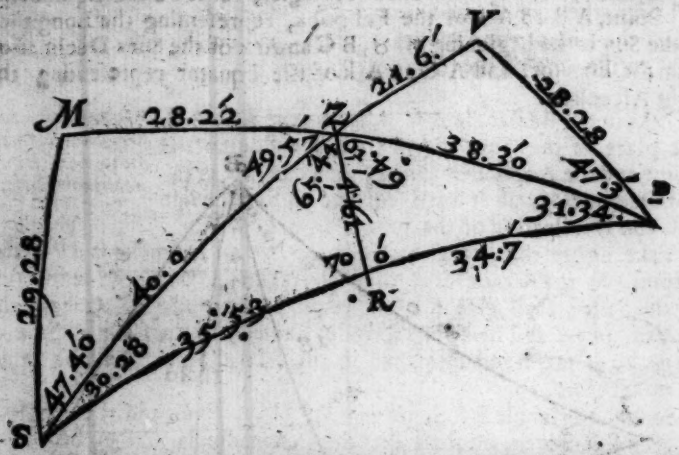


Let B D be an Ark of the Horizon, representing the Amplitude of the Suns rising from the East, and B E an Ark of the Horizon for his setting from the West: so D C shall be the difference of Ascension, and C E the difference of Descension; A D the Oblique Ascension, and A E the Oblique Descension of the same place of the Sun in our Latitude at *Oxford* of 51 *gr.* 45 *m.* whose Complement 38 *gr.* 15 *m.* is the Angle at E and D. The Triangles A C B, D C B, E C B, are Rectangled in C: the other A D B, A E B, consist every way of Oblique Angles.

Or, to fit an Example nearer to the Latitude of *London*. Let Z P S represent the Zenith, Pole, and Sun, Z P being 38 *gr.* 30 *m.* the Comple-

ment

ment of the Latitude, P S 70 *gr.* the Complement of the Declination, and Z S 40 *gr.* the Complement of the Suns Altitude. The Angle at Z shall shew the Azimuth, and the Angle at P, the Hour of the Day from the Meridian. Then if from Z to P S we let down a Perpendicular Z R, we shall reduce the Oblique Triangle into two Rectangle Triangles Z R P, Z R S. Or if from S to Z P we let down a Perpendicu-



lar S M, we shall reduce the same Z P S into two other Triangles, S M Z, S M P, Rectangled at M: whatsoever is said of any of these Triangles, the same holdeth for all other Triangles in the like cases.

For the Resolution of each of these, there be several ways. I only chuse those which are fittest for the *Sector*, wherein if that be remembred which before is shewed in the general use of the *Sector* concerning Lateral and Parallel entrance, it may suffice only to set down the Proposition of the three parts given, to the fourth required, and so shew first by the *Sines* alone.

In

In a Rectangled Triangle.

1. *To find a side, by knowing the Base, and the Angle opposite to the required side.*

As the Radius
is to the Sine of the Base:
So the Sine of the opposite Angle
to the Sine of the side required.

* As in the Rectangle A C B, having the Base A B, the place of the Sun 30 *gr.* from the Equinoctial Point, and the Angle B A C of 23 *gr.* 30 *m* the greatest Declination, if it were required to find the side B C the Declination of the Sun.

* *In Rectangled Triangles, the side opposite to the Rectangle is called the Base.*

Take either the lateral Sine of 29 *gr.* 30 *m.* and make it a Parallel Radius; so the Parallel Sine of 30 *gr.* taken and measured in the side of the *Sector*, shall give the side required 11 *gr.* 30 *m.* Or take the Sine of 30 *gr.* and make it a Parallel Radius; so the Parallel Sine of 23 *gr.* 30 *m.* taken and measured in the lateral Sines, shall be 11 *gr.* 30 *m.* as before.

So in the Triangle Z P S, having Z P 38 *gr.* 30 *m.* and the Angle P 31 *gr.* 34 *m.* given, we shall find the Perpendicular Z R to be 19 *gr.* 1 *m.* or having P S 70 *gr.* and the said Angle P 31 *gr.* 34 *m.* given, we may find the Perpendicular S M to be 29 *gr.* 28 *m.*

2. *To find the side by knowing the Base and the other side.*

As the Sine of the Complement of the side given
is to the Radius:
So is the Sine of the Complement of the Base
to the Sine of the Complement of the side required.

So in the Rectangle A C B, having A B 30 *gr.* and B C 11 *gr.* 30 *m.* given, the side A C will be found 27 *gr.* 54 *m.*

Or in the Rectangle Z R P, having Z P 38 *gr.* 30 *m.* and Z R 29 *gr.* 1 *m.* given, the side R P will be found 34 *gr.* 7 *m.*

3. To

3. *To find a side, by knowing the two Oblique Angles.*

As the Sine of either Angle
to the Sine of the Complement of the other Angle:
So is the Radius
to the Sine of the Complement of the side opposite to the second Angle.

So in the Rectangle A C B having C A B for the first Angle 23 *gr.* 30 *m.* and A C B, for the second 69 *gr.* 22 *m.* the side A C will be found 27 *gr.* 54 *m.* Or making A B C the first Angle, and C A B the second, the side B C will be found 11 *gr.* 30 *m.*

4. *To find the Base, by knowing both the sides.*

As the Radius
to the Sine of the Complement of the one side:
So the Sine of the Complement of the other side,
to the Sine of the Complement of the Base required.

So in the Rectangle A C B having A C 27 *gr.* 54 *m.* and B C 11 *gr.* 30 *m.* the Base A B will be found 30 *gr.*

5. *To find the Base by knowing the one side, and the Angle opposite to that side.*

As the Sine of the Angle given,
to the Sine of the side given:
So is the Radius
to the Sine of the Base required.

So in the Rectangle B C D, knowing the Latitude and the Declination, we may find the Amplitude; as having B C the side of the Declination 11 *gr.* 30 *m.* and B D C the Angle of the Complement of the Latitude 38 *gr.* 15 *m.* the Base B D, which is the Amplitude, will be found to be 18 *gr.* 47 *m.*

6. *To*

6. To find an Angle, by the other Oblique Angle, and the side opposite to the inquired Angle.

As the Radius,
to the Sine of the Complement of the side:
So the Sine of the Angle given,
to the Sine of the Complement of the Angle required.

So in the Rectangle A C B, having the Angle B A C 23 *gr.* 30 *m.* and the side A C 27 *gr.* 54 *m.* the Angle A B C will be found 69 *gr.* 21 *m.*

7. To find an Angle, by the other Oblique Angle, and the side opposite to the Angle given.

As the Sine of the Complement of the side
to the Sine of the Complement of the Angle given:
So is the Radius,
to the Sine of the Angle required.

So in the Rectangle A C B, having B A C 23 *gr.* 30 *m.* and B C 11 *gr.* 30 *m.* the Angle A B C will be found 69 *gr.* 21 *m.*

8. To find an Angle, by the Base, and the side opposite to the inquired Angle.

As the Sine of the Base
is to the Radius:
So the Sine of the side
to the Sine of the Angle required.

So in the Rectangle B C D, having B D 18 *gr.* 47 *m.* and B C 11 *gr.* 30 *m.* the Angle B D C will be found 38 *gr.* 15 *m.*

These eight Propositions have been wrought by the Sines alone; those which follow require joynt help of the Tangent.

And forasmuch as the Tangent could not well be extended beyond 63 *gr.* 30 *m.* I shall set down two ways for the resolution of each Proposition; so that if the one will not hold, the other may.

9. *To find a side, by having the other side, and the Angle opposite to the inquired side.*

1. As the Radius
to the Sine of the side given:
So the Tangent of the Angle,
to the Tangent of the side required.

2. As the Sine of the side given,
is to the Radius:
So the Tangent of the Complement of the Angle
to the Tangent of the Complement of the side required.

So in the Rectangle A C B, having the side A C 27 *gr.* 54 *m.* and the Angle B A C 23 *gr.* 30 *m.* the side B C will be found to be 11 *gr.* 30 *m.*

10. *To find a side, by having the other side, and the Angle next the inquired side.*

As the Tangent of the Angle,
to the Tangent of the side given:
So is the Radius
to the Sine of the side required.

2. As the Tangent of the Complement of the side,
to the Tangent of the Complement of the Angle:
So is the Radius
to the Sine of the side required.

This and the like, where the Tangent standeth in the first place, are best wrought by Parallel entrance. And so in the Rectangle B C D, having B C the side of Declination 11 *gr.* 30 *m.* and B D C the Angle of the Complement of the Latitude 38 *gr.* 25 *m.* the side D C, which is the Ascensional difference, will be found 14 *gr.* 57 *m.*

By the Ascensional difference is given the time of the Suns rising and setting, and length of the day: allowing an hour for each 15 *gr.* and four minutes of time for each several degree. As in the example, the difference between the Suns Ascension in a right Sphere, which is always

always at ſix of the clock, and his Aſcenſion in our Latitude being 14 *gr.* 57 *m.* it ſheweth that the Sun riſeth very near an hour before ſix, becauſe of the Northern Declination; or after ſix, if the Sun be declining to the Southward.

11. *To find a ſide by knowing the Baſe, and the Angle adjacent next to the inquired ſide.*

1. As the Radius,
To the Sine of the Complement of the Angle:
So the Tangent of the Baſe,
to the Tangent of the ſide required.

2. As the Sine of the Complement of the Angle,
is to the Radius:
So is the Tangent of the Complement of the Baſe,
to the Tangent of the Complement of the ſide required.

So in the Rectangle A C B, knowing the place of the Sun from the next Equinoctial Point, and the Angle of his greateſt Declination, we may find his right Aſcenſion: *viz.* the Baſe A B 30 *gr.* and the Angle B A C 23 *gr.* 30 *m.* being given, the right Aſcenſion A C will be found 27 *gr.* 54 *m.*

12. *To find the Baſe by knowing the Oblique Angles.*

As the Tangent of the one Angle,
to the Tangent of the Complement of the other Angle:
So is the Radius,
to the Sine of the Complement of the Baſe.

So in the Rectangle A C B, having B A C 23 *gr.* 30 *m.* and A B C 69 *gr.* 21 *m.* the Baſe A B will be found 30 *gr.*

13. *To find the Baſe, by one of the ſides, and the Angle adjacent next that ſide.*

1. As the Radius,
is to the Sine of the Complement of the Angle:

So the Tangent of the Complement of the side,
to the Tangent of the Complement of the Base.

2. As the Sine of the Complement, of the Angle,
is to the Radius:
So the Tangent of the side given,
to the Tangent of the Base required.

So in the Rectangle A C B, having A C 27 *gr.* 54 *m.* and B A C 23 *gr.* 30 *m.* the Base A B will be found 30 *gr.* 00 *m.*

14. *To find an Angle, by knowing both the sides.*

1. As the Radius,
is to the Sine of the side next the inquired Angle:
So the Tangent of the Complement of the oppolite side,
to the Tangent of the Complement of the Angle required.

2. As the Sine of the side next the inquired Angle,
is to the Radius:
So the Tangent of the oppolite side,
to the Tangent of the Angle required.

So in the Rectangle A C B, having A C 27 *gr.* 54 *m.* and B C 11 *gr.* 30 *m.* the Angle at A will be found 23 *gr.* 30 *m.* and the Angle at B 69 *gr.* 21 *m.*

15. *To find the Angle, by the Base, and the side adjacent to the required Angle.*

1. As the Tangent of the Complement of the side,
to the Tangent of the Complement of the Base:
So is the Radius,
to the Sine of the Complement of the Angle required.

2. As the Tangent of the Base,
to the Tangent of the side:
So is the Radius,
to the sign of the Complement of the Angle required.

So in the Rectangle B C D, having the Base B D 16 *gr.* 47 *m.* and the side B C 12 *gr.* 30 *m.* the Angle D B C between them will be found 53 *gr.* 15 *m.*

16. *To find an Angle, by knowing the other Oblique Angle, and the base.*

1. As the Radius,
to the Sine of the Complement of the Base:
So the Tangent of the Angle given,
To the Tangent of the Complement of the Angle required.

2. As the Sine of the Complement of the Base,
is to the Radius:
So the Tangent of the Complement of the Angle given,
to the Tangent of the Angle required.

So in the Rectangle A C B, having the Angle at A 23 *gr.* 30 *m.* and the Base A B 30 *gr.* the Angle A B C will be found 69 *gr.* 12 *m.*

These sixteen Cases are all that can fall out in a Rectangle Triangle: those which follow do hold

In any Spherical Triangle whatsoever.

17. *To find a side, opposite to an Angle given, by knowing one side, and two Angles, whereof one is opposite to the given side, the other to the side required.*

As the Sine of the Angle opposite to the side given,
is to the Sine of the side given:
So the Sine of the Angle opposite to the side required,
to the Sine of the side required.

So in the Triangle A B E, having the place of the Sun, the Latitude, and the greatest Declination, we may find the Amplitude. As having A B 30 *gr.* B A E 23 *gr.* 30 *m.* and A B E 38 *gr.* 15 *m.* the side B E which is the Amplitude, will be found 18 *gr.* 47 *m.*

18. To

18. *To find an Angle opposite to a side given, by having one Angle and two sides, the one opposite to the given Angle, the other to the Angle required.*

As the Sine of the side opposite to the Angle given,
is to the Sine of that Angle given:
So the Sine of the side opposite to the Angle required,
to the Sine of the Angle required.

So in the Triangle Z P S, having the Azimuth, and Latitude, and Declination, we may find the hour of the day. As having P Z S 130 *gr.* 3 *m.* P S 70 *gr.* and Z S 40 *gr.* the Angle Z P S, which sheweth the hour from the Meridian, shall be found 31 *gr.* 34 *m.*

19. *To find an Angle by knowing the three sides.*

This Proposition is most useful, but most difficult of all others: as in Arithmetick, so by the *Stellar*, yet may it be performed several ways.

1. According to *Regiomontanus* and others.

As the Sine of the lesser side, next the Angle required,
to the difference of the versed Sines of the Base, and difference of
So is the Radius, (the sides:
to a fourth proportional.

Then as the Sine of the greater side next the Angle required,
is to that fourth proportional:
So is the Radius,
to the versed Sine of the Angle required.

So in the Triangle Z P S, having the side P S, the Complement of the Declination 70 *gr.* 0 *m.* the side Z P the Complement of the Latitude 38 *gr.* 30 *m.* and the Base Z S the Complement of the Altitude 40 *gr.* the Angle of the hour of the day Z P S will be found 31 *gr.* 34 *m.* which is 2 *h.* 6 *m.* from the Meridian.

For the Base being 40 *gr.* 0 *m.* and the difference of the sides 38 *gr.* 30 *m.* and 70 *gr.* 0 *m.* being 31 *gr.* 30 *m.* the difference of their versed Sines

Sines will be the ſame with the diſtance between the right Sine of 50 *gr.* and 58 *gr.* 30 *m.* This difference I take out, and make it a Parallel Sine of the leſſer ſide 38 *gr.* 30 *m.* ſo the Parallel Radius will be the fourth proportional. Then coming to the ſecond operation, I make this fourth proportional a Parallel Sine of the greater ſide of 70 *gr.* 0 *m.* and take out this Parallel Radius. For this meaſured from 90 *gr.* toward the Center, will be the verſed Sine of 31 *gr.* 34 *m.*

In the like ſort in the ſame Triangle Z P S, having the ſame Complements given, the Angle P Z S which is the Azimuth from the North part of the Meridian, will be found 130 *gr.* 3 *m.* For here the Baſe oppoſite to the Angle required being 70 *gr.* and the difference of the ſides 38 *gr.* 30 *m.* and 40 *gr.* being 1 *gr.* 30 *m.* the difference of their verſed Sines will be the ſame with the diſtance between the right Lines of 20 *gr.* and 88 *gr.* 30 *m.* This difference I take and make it a Parallel Sine of the leſſer ſide 38 *gr.* 30 *m.* ſo the Parallel Radius will be the fourth proportional. Then coming to the ſecond operation, I make this fourth proportional a Parallel Sine of the greater ſide 40 *gr.* and take out this Parallel Radius; for this meaſured from 90 *gr.* beyond the Center, in the Lines of *Sines*, ſtretched forth at their full length, will be the verſed Sine of 130 *gr.* 3 *m.*

2. I may find an Angle by knowing three ſides, by that which I have elſewhere demonſtrated upon *Barth. Pitiſcus*, and that at one operation in this manner.

At the Sine of the greater ſide,
is to the Secant of the Complement of the other ſide:
So the differerence of Sines of the Complement of the Baſe,
and the Ark compounded of the leſſer ſide with the
Complement of the greater,
to the verſed Sine of the Angle required.

So in the ſame Triangle Z P S, having the ſame Complements given, the Angle at P, which ſheweth the hour from the Meridian, will be found as before, 31 *gr.* 34 *m.*

For the ſides being 38 *gr.* 30 *m.* and 70 *gr.* 0 *m.* I take the Secant of the Complement of 38 *gr.* 30 *m.* and make it a Parallel Sine of 70 *gr.* then keeping the Sector at this Angle, I conſider that the Complement of 70 *gr.* being 20 *gr.* added unto 38 *gr.* 30 *m.* the compounded ſide (which is here the Meridian Altitude) will be 58 *gr.* 30 *m.*

30 *m.* and that the Base being 40 *gr.* the difference of Sines of the compounded side, and the Complement of the Base will be (as before) the distance between the Sines of 50 *gr.* and 58 *gr.* 30 *m.* Wherefore I take out this difference, and lay it on both the Lines of *Sines* from the Center: so the Parallel taken in the terms of this difference, and measured from 90 *gr.* towards the Center, doth give the versed Sine of 31 *gr.* 34 *m.*

This example of finding the hour of the day might otherwise have been proposed in these terms,

As the Sine of the Complement of the Declination,
is to the Secant of the Latitude:
So the differerence between the Sine of the Altitude proposed,
and the Sine of the Meridian Altitude,
to the versed Sine of the hour from the Meridian:

Then the Latitude being 51 *gr.* 30 *m.* the Declination 20 *gr.* Northward, and the Altitude 50 *gr.* the work would be the same as before.

The other Angles P Z S, P S Z, may be found in the same sort; but having the sides and one Angle, it will be sooner done by that which we shewed before in the 18 *Prop.*

20. *To find a side, by knowing the three Angles.*

If for the greater Angle, we take his Complement to 180 *gr.* the Angles shall be turned into sides, and the sides into Angles, and the operation shall be the same, as in the former *Prop.*

As in the Triangle Z P S, having the Angle Z P S 31 *gr.* 34 *m.* Z S P 30 *gr.* 28 *m.* and P Z S 130 *gr.* 3 *m.* I would take the greater Angle of 130 *gr.* 3 *m.* out of 180 *gr.* and there remains 49 *gr.* 57 *m.* Then as I had a Triangle of three known sides, one of 31 *gr.* 34 *m.* another of 30 *gr.* 20 *m.* and a third of 49 *gr.* 57 *m.* I would seek the Angle opposite to one of these sides, by the last *Prop.* So the Angle which is thus found, would be the side, which is here required.

21. *To find a side, by having the other two sides, and the Angle comprehended.*

This Proposition being the converse of the nineteenth, may be wrought

wrought accordingly: but the beſt way both for it and thoſe which follow, is to reſolve them into two Rectangles, by letting down a Perpendicular, as was ſhewed in the firſt *Prop.*

So in the Triangle Z P S, having Z P the Complement of the Latitude, and P S the Complement of the Declination, with Z P S the Angle of the hour from the Meridian, we may find Z S the Complement of the Altitude of the Sun.

For having let down the Perpendicular Z R, by the firſt *Prop.* we have two Triangles, Z R P, Z R S, both rectangled at R. Then may we find the ſide P R, either by the ſecond, or tenth, or eleventh *Prop.* which taken out of P S, leaveth the ſide R S: with this R S and Z R we may find the Baſe Z S by the fourth *Prop.*

Or having let down the Perpendicular S M, we have two Rectangle Triangles S M Z, S M P. Then may we find M P, from which if we take Z P, there remaineth M Z: but with M Z and S M, we may find the Baſe Z S.

22. *To find a ſide, by having the other two ſides, and one of the Angles next the inquired ſide.*

So in the Triangle Z P S, having Z P, the Complement of the Latitude, and P S the Complement of the Declination, with P Z S the Angle of the Azimuth, we may find Z S the Complement of the Altitude of the Sun.

For having Z P, and the Angle at Z, we may to S Z produced, let down a Perpendicular P V. Then we have two Rectangle Triangles P V Z, P V S, wherein if we find the ſides V Z, V S, and take the one out of the other, there will remain the ſide required Z S.

23. *To find a ſide, by having one ſide, and the two Angles next the inquired ſide.*

So in the Triangle A B D, having A B the place of the San, and B A D the Angle of the greateſt Declination, and A D B the Angle of the Equator with the Horizon, we may find A D the Oblique Aſcenſion.

For having let down B C the Perpendicular of Declination, we have two Rectangled Triangles, A C B, D C B. Then may we find A C the

right Aſcenſion, and D C the aſcenſional difference; and comparing the one with the other, there remaineth A D.

24 *To find a ſide, by having two Angles, and the ſide incloſed by them.*

So in the Triangle Z P S, having the Angles at Z and P, with the ſide intercepted Z P, we may find the ſide P S. For having let down the Perpendicular P V, we have two Rectangles P V Z, P V S. Then may we find the Angle V P Z, either by the ſeventh, or fifteenth, or ſixteenth *Prop.* which added to Z P S, maketh the Angle V P S, with this V P S, and P V, we may find the Baſe P S, according to the 13 *Prop.*

25. *To find an Angle by having the other two Angles and the ſide incloſed by them.*

So in the Triangle Z P S, having the Angles at Z and P, with the ſide intercepted Z P, we may find the other Angle Z S P. For having let down the Perpendicular Z R, we have two Rectangles Z R P, Z R S. Then may we find the Angle P Z R by the ſixteenth *Prop.* and that compared with P Z S, leaveth the Angle R Z S: with this R Z S, and Z R, we may find the Angle required Z S R, according to the ſixth *Propoſition.*

26. *To find an Angle, by having the other two Angles, and one of the ſides next the inquired Angle.*

So in the Triangle A B D, having the Angles at A and D, with the ſide A B, we may find the Angle A B D. For having let down the Perpendicular B C, we have two Rectangles, A C B, D C B. Then may we find the Angles A B C, D B C, and take D B C out of A B C; for ſo there remaineth the Angle required A B D.

27. *To find an Angle, by knowing two ſides, and the Angle contained by them.*

So in the Triangle Z P S, having the ſides Z P, P S, with the Angle comprehended Z P S, we may find the Angle P Z S. For having let down the Perpendicular S M, we have two Rectangles S M Z, S M P.

Then

Then may we find the ſide M P, and taking Z P out of M P, there remaineth M Z: with this M Z and the Perpendicular M S, we may find the Angle M Z S, by the fourteenth *Prop.* This Angle M Z S, taken out of 180 *gr.* there remaineth P Z S.

28. *To find an Angle, by knowing the two ſides next it and one of the other Angles.*

So in the Triangle Z P S, having the ſides Z P, and P S, with the Angle P Z S, we may find the Angle Z P S; For having let down the Perpendicular P V, we have two Rectangles P V Z, P V S. Then may we find the Angles V P Z, V P S: and taking V P Z out of V P S, there remaineth Z P S, which was required.

Theſe 28 Caſes are all that can fall out in any Spherical Triangle: if any do not preſently underſtand them, let them once more read over the uſe of the Globes, and they ſhall ſoon become eaſie unto them.

CHAP. VI.

Of the Uſe of the Meridian Line in Navigation.

THe Meridian Line is here ſet on the ſide of the Sector ſtretched forth at full length, on the ſame Plane with the Line of *Lines* and *Solids*, and is divided unequally toward 87 *gr.* (whereof 70 *gr.* are about one half) in ſuch ſort as the Meridian in the Chart of *Mercators* Projection. The Uſe of it may be,

1. *To divide a Sea-chart according to* Mercators *Projection.*

If a degree of the Equator on the Sea-chart, be equal to the hundred part of the Line of *Lines* in the *Sector*, the degrees of the Meridian upon the *Sector*, ſhall give the like degrees upon the Sea-chart: if otherwiſe they be unequal, then may the Meridians of the Sea-chart be divided in ſuch ſort as the Line of Meridians is divided on the *Sector*, by that which we ſhewed before in the 8 *Prop.* of the Line of Lines.

But to avoid error, I have here ſet down a Table, whereby the Meridian Line may be divided out of the degrees of the Equator ſuppoſing

each degree in the Equator, to be ſubdivided into a thouſand parts. By which Table, and the uſual Table of Sines, Tangents, and Secants, the Proportions following may be alſo reſolved Arithmetically. For the manner of diviſion, let the Equator be drawn, and divided, and croſſed with Parallel Meridians, as in the common Sea-chart: then look into the Table, and let the diſtance between the Equator and 40 *gr.* in the Meridian, from the Equator, be equal to 43 *gr.* 711 *parts* of the Equator, as in the Table: let 50 *gr.* in the Meridian from the Equator, be equal to 57 *gr.* 909 *parts* of the Equator, and ſo in the reſt.

The making of this Table is, by addition of Secants. For the Parallels of Latitudes being leſs than the Equator or Meridian in ſuch proportion as the Radius is to the Secant of the Parallel. For example, the Parallel of 60 *degrees* of Latitude is leſs than the Equator (and conſequently, each degree of this Parallel of 60 degrees leſs than a degree of the Equator, or Meridian) in ſuch proportion as 100000 the Radius, hath unto 200000 the Secant of 60 *degrees*.

M	Gr. Par.	M	Gr. Par.	M	Gr. Par.	M	Gr. Par.	M	Gr. Par.
0	0. 0	4	3.001	6	6.011	9	9.037	12	12.088
	.100		3.101		6.111		9.138		12.190
	.200		3.201		6.212		9.239		12.293
	.300		3.301		6.312		9.341		12.395
	.400		3.402		6.413		9.442		12.497
	.500		3.502		6.514		9.543		12.600
	.600		3.602		6.614		9.645		12.702
	.700		3.702		6.715		9.740		12.805
	.800		3.803		6.816		9.848		12.907
	.900		3.903		6.916		9.949		13.010
1	1.000	4	4.003	7	7.017	10	10.051	13	13.112
	1.100		4.103		7.118		10.152		13.215
	1.200		4.204		7.219		10.254		13.318
	1.300		4.304		7.319		10.355		13.421
	1.400		4.404		7.420		10.457		13.523
	1.500		4.504		7.521		10.559		13.626
	1.600		4.605		7.622		10.661		13.729
	1.700		4.705		7.723		10.762		13.832
	1.800		4.805		7.824		10.864		13.935
	1.900		4.906		7.925		10.966		14.038
2	2.000	5	5.006	8	8.026	11	11.068	14	14.141
	2.100		5.106		8.127		11.170		14.244
	2.200		5.207		8.228		11.272		14.347
	2.300		5.307		8.329		11.374		14.450
	2.400		5.408		8.430		11.476		14.553
	2.500		5.508		8.531		11.578		14.656
	2.601		5.609		8.632		11.680		14.760
	2.701		5.709		8.733		11.782		14.863
	2.801		5.810		8.834		11.884		14.967
	2.901		5.910		8.936		11.986		15.070
3	3.001	6	6.011	9	9.037	12	12.088	15	15.174

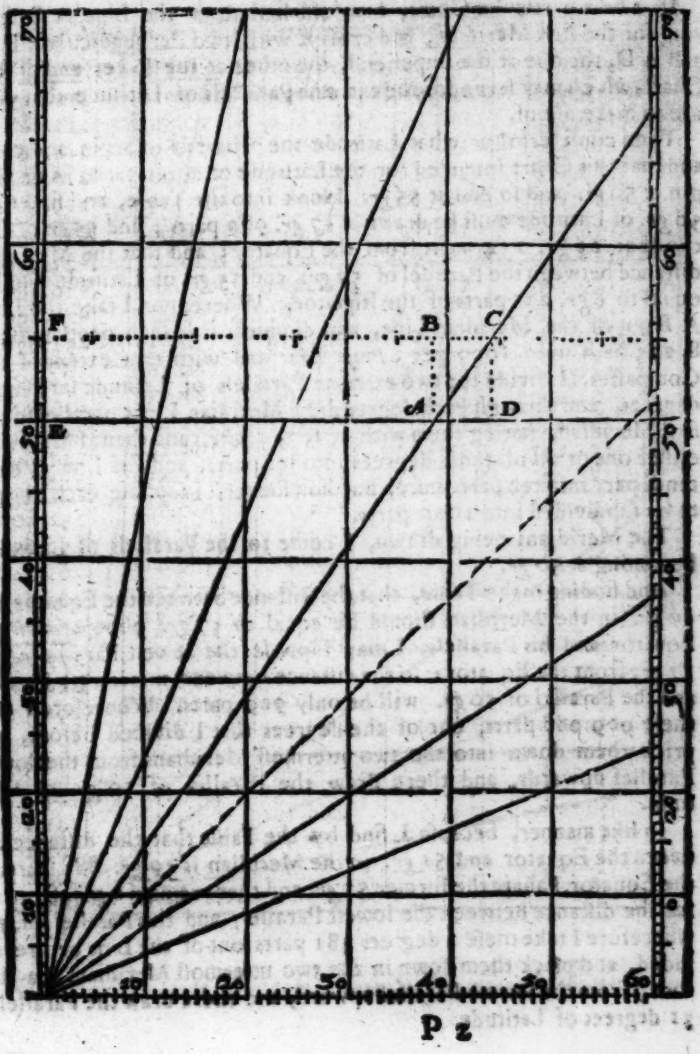
M	Gr. Par.	M	Gr. Par.	M	Gr. Par	M	Gr. Par.	M	Gr. Par.
15	15.174	18	18.303	21	21 486	24	24.734	27	28.058
	15.277		18.408		21.503		24.844		28.171
	15.381		18 513		21.70		24.953		28.283
	15.485		18.619		21.808		25.063		28.396
	15 588		18.724		21.915		25.173		28.508
	15.692		18.830		21.023		25 282		28.621
	15.796		18 935		22.130		25.392		28.734
	15 900		19.041		22.238		25.502		28.847
	16.004		19.146		22.345		25 613		28.959
	16.107		19.251		22.453		25.723		29.072
16	16.211	19	19.356	22	22.561	25	25.833	28	29.186
	16.316		19.463		22.669		25.943		29.299
	16.420		19.569		22.777		26.054		29.313
	16.524		19.675		22.885		26.164		29.526
	16.628		19.781		22.993		26.275		29.640
	16.732		19.887		23.101		26.386		29.753
	16.836		19.693		23.210		26.497		29.867
	16.941		20.100		23.318		26.608		29.981
	17.045		20.206		23.427		26.719		30.095
	17.150		20.312		23.535		26.830		30.209
17	17.255	20	20.419	23	23.643	26	26.941	29	30.324
	17.359		20.525		23.752		27.052		30.438
	17.464		20.632		23.861		27.164		30.553
	17.568		20 738		23.970		27.275		30.667
	17.673		20.845		24.079		27.387		30.782
	17.778		20.951		24.188		27.499		30.897
	17.883		21.059		24 297		27.610		31.012
	17.988		21.165		24.406		27.722		31.127
	18.093		21.292		24.515		27.834		31.242
	18.198		21.379		24.624		27.946		31.357
18	18.302	21	21.486	24	24.734	27	28.058	30	31.473

M	Gr. Par	M	Gr. Par.	M	Gr. Par	M	Gr. Par.	M	Gr. Par.
30	31.473	33	34.992	36	38 633	39	42.415	42	46 362
	31.885		35.111		38.757		42.544		46.496
	31.704		35.231		38 880		42.673		46.631
	31.820		35.350		39 004		42.802		46 766
	31.936		35.470		39 129		42.931		46.902
	32.052		35.590		39.853		43.061		47.037
	32.168		31.710		39.377		43 291		47.173
	32.284		35.830		39.502		43.320		47.309
	32.409		35.950		39.627		43.451		47.440
	32.516		36.071		39.752		43.581		47.581
31	32.633	34	36.191	37	39.877	40	43.711	23	47.718
	32.750		36.312		40.002		43.842		47.855
	32.867		36.433		40.128		43.973		47 992
	32.984		36.554		40.253		44.104		48.129
	33.101		36.675		40.379		44.235		48.267
	33.218		36.796		40 505		44.366		48.404
	33.336		36.917		40 631		44.498		48.542
	33.453		37.039		40.757		44 630		48.681
	33.571		37.161		40 884		44.762		48.819
	33.688		37.282		41.014		44 894		48.958
32	33.806	35	37.405	38	41.137	41	45.026	44	49.079
	33.924		37.527		41.264		45.159		49.236
	34.042		37.643		41.322		45.292		49.375
	34.161		37.771		41.519		45.425		49.515
	34.279		37.894		41.646		45.558		49.655
	34.397		38.017		41.774		45.691		49.795
	34.516		38.140		41.902		45.825		49.935
	34.635		38 263		42.030		45.959		50 076
	34.754		38.386		42.158		46.093		50.217
	34.873		38.509		42.287		46.227		50.358
33	34.992	36	38.633	39	42.415	42	46.362	45	50.499

M	Gr. Par.	M	Gr. Par.	M	Gr. Par.	M	Gr. Par.	M	Gr. Par.
45	50.499	48	54.860	51	59 481	54	64.412	57	69.711
	50.641		55.010		59.640		64.582		69.895
	50.783		55.160		59.800		64.753		70.080
	50.925		55.310		59 960		64.924		70 265
	51.068		55.460		60.120		65.096		70.449
	51.210		55.611		60.280		65.268		70.635
	51.353		55.762		60.441		65 440		70 821
	51.496		55.913		60.601		65.613		71.008
	51.639		56.065		60.763		65.786		71.195
	51.783		56.217		60.923		65.960		71.483
46	51.927	49	56.369	52	61.088	55	66.134	58	71.572
	52.071		56.522		61.250		66.308		71.701
	52.215		56.675		61.413		66.483		71 950
	52.360		56.828		61.577		66.659		72.140
	52.505		56.981		61.741		66.835		72 331
	52.650		57.135		61.904		87.011		72.522
	52.795		57.289		62 069		67.188		72.714
	52.941		57.444		62.234		67 365		72.906
	53.087		57.598		62.399		67.543		73.099
	53.233		57.704		62.564		67.721		73.292
47	53.380	50	57.909	53	62.730	56	67.900	59	73 486
	53. 26		58 065		62.897		68.079		73.680
	53.073		58.221		63.063		68.258		73 875
	53.821		58.377		63.231		68.438		74.071
	53.968		58.534		63.398		68 618		74.267
	54.116		58 691		63.566		68.799		74.464
	54.264		58 848		63.734		68.981		74.661
	54.413		59.006		63.903		69.163		74 859
	54.562		59.164		64 072		69.345		75.057
	54.711		59.322		64.042		69.528		75.256
48	54.860	51	59.481	54	64 412	57	69.712	60	75.456

M	Gr. Par.	M	Gr. Par.	M	Gr. Par.	M	Gr. Par.	M	Gr. Par.
60	75.451	63	81.749	66	88.725	69	96.575	72	105.579
	75.656		81.970		88.971		96.854		105.904
	75.853		82.191		89.219		97.135		106.230
	76.057		82.413		89.467		97.418		106.558
	76.261		82.635		89.716		97.701		106.888
	76.464		82.860		89.967		97.986		107.220
	76.667		83.084		90.218		98.272		107.553
	76.871		83.313		90.470		98.560		107.888
	77.076		83.536		90.723		98.849		108.226
	77.281		83.763		90.978		99.139		108.665
61	77.487	64	83.990	67	91.232	70	99.431	73	108.906
	77.694		84.219		91.489		99.724		109.249
	77.901		84.448		91.746		100.018		109.594
	78.109		84.678		92.005		100.314		109.941
	78.367		84.909		92.264		100.612		110.290
	78.526		85.141		92.525		100.910		110.641
	78.736		85.374		92.787		101.211		110.994
	78.947		85.607		93.050		101.513		111.349
	79.158		85.842		93.314		101.816		111.707
	79.370		86.077		93.579		102.121		112.066
62	79.583	65	86.313	68	93.846	71	102.427	74	112.428
	79.796		86.550		94.213		102.735		112.792
	80.010		86.788		94.382		103.044		113.158
	80.225		87.027		94.652		103.356		113.526
	80.441		87.267		94.923		103.668		113.897
	80.657		87.508		95.195		103.983		114.270
	80.874		87.749		95.468		104.299		114.645
	81.091		87.992		95.743		104.616		115.023
	81.310		88.235		96.019		104.936		115.403
	81.529		88.480		96.296		105.257		115.786
63	81.749	66	88.725	69	96.575	72	105.579	75	116.171

M	Gr. Par.	M	Gr. Par.	M	Gr. Par.	M	Gr. Par.	M	Gr. Par.
75	116.171	78	129.075	81	145.650	84	168.947	87	208.705
	116.359		129.538		146.292		169.912		210.649
	116.949		130.065		146 942		170.893		212.668
	117.342		130.536		147.600		171.891		214.745
	117.737		131.031		148.265		172.907		216.909
	118.135		131.530		148.937		173.941		219.158
	118.536		132.034		149.618		174 994		221.498
	118.939		132.542		150.307		176.067		223.938
	119.345		133.055		151.003		177.160		226.486
	119.755		133.572		151.709		178.275		229.153
76	120.166	79	134.094	82	152 423	85	179.411	88	231.950
	121.581		134.620		153.147		180.569		234.891
	121.000		135.151		153.878		181.752		237.991
	121.420		135.687		154.620		182.960		241.268
	121.843		136.228		155.372		184.194		244 744
	122.270		136.775		156.132		185.454		248.445
	122.700		137.326		156.903		186.743		252.402
	123.133		137.883		157.685		188.062		256 652
	123.570		138.445		158.478		189.411		261.243
	124.009		139.012		159.281		190.793		266.235
77	124.452	80	139.585	83	160.096	86	192 210	89	271.705
	124.898		140.164		160.922		193 661		277.753
	125.348		140.748		161.761		195 151		284.517
	125.801		141.339		162.612		196.680		292.191
	126.258		141.936		163.474		198.251		301.058
	126.718		142.538		164 352		199.867		311.563
	127.182		143.147		165 243		201.529		324 455
	127.649		143.763		166.146		203.240		341.166
	128.121		144.385		167.065		205.005		365.039
	128.596		145.014		167.999		206.825		408.011
78	129.075	81	145 650	84	168.947	87	208.705	90	Infinite.


F
B
C
A
D
E
P z

If it be a particular Chart, I would first draw the Line A B serving for the first Meridian, and cross it with two Perpendiculars B C and A D, the one at the upper end, the other at the lower end of the Chart, which may serve for the extreme Parallels of Latitude that you are to make use of.

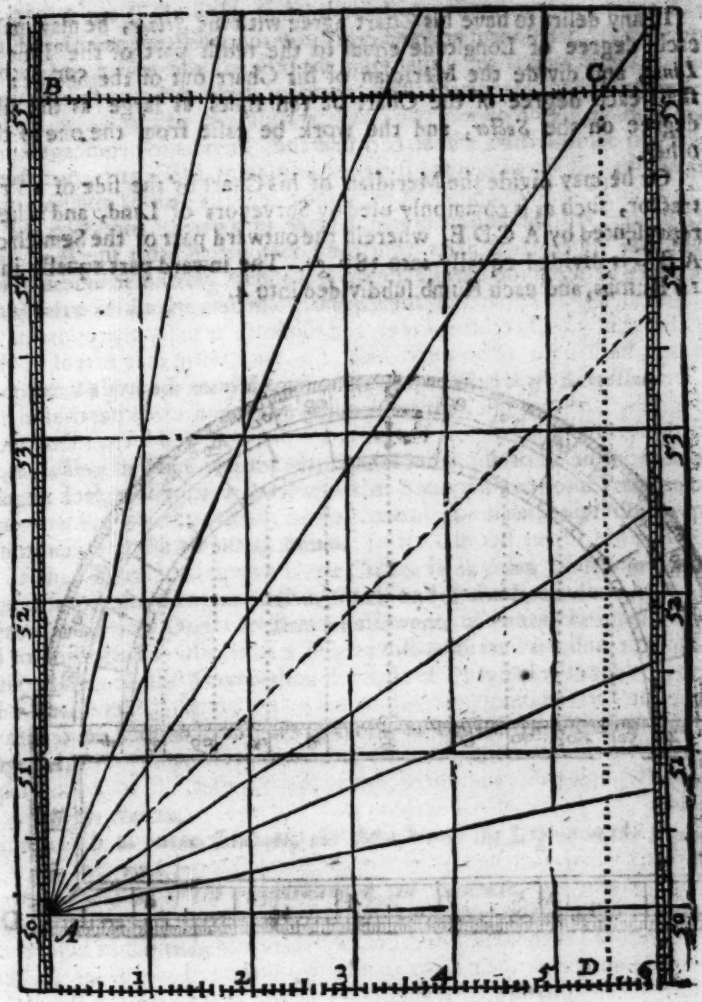
Then considering at what Latitude the Chart is to begin and end, and that this Chart intended for the Latitude of these parts, is to begin at 50 *gr.* and so end at 55 *gr.* I look into the Table, and find that 50 *gr.* of Latitude must be drawn at 57 *gr.* 909 parts; and 55 *gr.* of Latitude at 66 *gr.* 134 parts from the Equator; and that the Meridian distance between the Parallel of 50 *gr.* and 55 *gr.* of Latitude must be equal to 8 *gr.* 225 parts of the Equator. Whereupon I take the Line A B out of the Meridian Line, and diminish it in such proportion as 8, 225 hath unto 1000 per 3 *Prop. Lin.* and with that extent of the Compasses, I divide the two extreme Parallels of Latitude into equal degrees, and through each degree draw Meridian Lines parallel to the first Meridian, noting them with 1, 2, 3, 4, *&c.* and then, I subdivide either one or all of those degrees into ten parts, and (if I may) each tenth part into ten parts more, but howsoever, I suppose each degree to be subdivided into 1000 parts.

The Meridians being drawn, I come to the Parallels of Latitude, beginning at 50 *gr.*

And finding in the Table, that the distance between the Equator and 50 *gr.* in the Meridian should be equal to 57 *gr.* 909 parts in the Equator and his Parallels, I may suppose the lowest Parallel to be 57 *gr.* from the Equator: so the distance between this lowest Parallel and the Parallel of 50 *gr.* will be only 909 parts. Wherefore I take these 909 odd parts, out of the degrees that I divided before, and prick them down into the two uttermost Meridians from the lowest Parallel upwards, and there draw the Parallel of 50 *gr.* of Latitude.

In like manner, because I find by the Table that the distance between the Equator and 51 *gr.* in the Meridian is 59 *gr.* 881 parts of the Equator, I abate the former 57 *gr.* and there remain 2 *gr.* 481 parts for the distance between the lowest Parallel, and this Parallel of 51: wherefore I take these 2 degrees 481 parts out of the Line before divided, and prick them down in the two uttermost Meridians (as before) from the lowest Parallel upward, and there draw the Parallel of 51 degrees of Latitude.

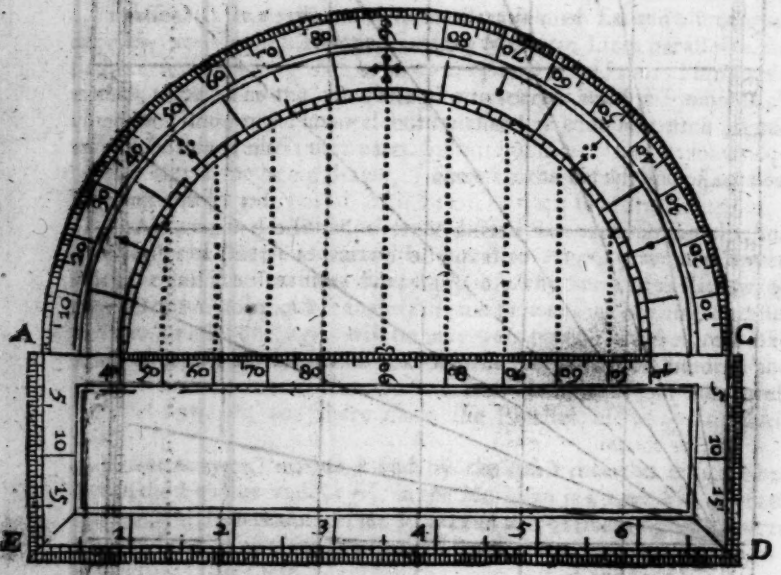
If



The

If any desire to have his Chart agree with his *Sector*, he may make each degree of Longitude equal to the tenth part of the Line of *Lines*, and divide the Meridian of his Chart out of the Sector: so shall each degree of the Chart be ten times as large as the like degree on the *Sector*, and the work be easie from the one to the other.

Or he may divide the Meridian of his Chart by the side of a Protractor, such as is commonly used by Surveyors of Land, and is here represented by A C D E, wherein the outward part of the Semicircle A B C is divided equally into 180 *gr.* The inward part equally into 16 Rumbs, and each Rumb subdivided into 4.



The

The Lines C D, D E, E A, divided equally according to the Line of Lines upon the *Sector*, or the Parallels upon the Chart. Onely the Diameter A C would be divided unequally, by letting down occult perpendicular Lines upon it from each Degree in the Semicircle, which being done, the intermediate part between the Rumbs and the Diameter may be all cut forth: And the back side of the long Square may be filled with 6 Lines of Chords, or Scales of several parts in the Inch.

So may the Meridian be divided by the parts of the Side E D, the Angles of each Rumb may readily be pricked down by the Degrees in the Semicircle, and the Line of Chords and the other Scales may serve to do the like with more variety.

2. *To find how many Leagues answer to one Degree of Longitude in every several Latitude.*

In sailing by the Compass, the Course holds sometime upon a Great Circle, sometime upon a Parallel to the Equator; but most commonly upon crooked Lines, winding towards one of the Poles, which Lines are well known by the Name of *Rumbs*.

If the Course hold upon a Great Circle, it is either North or South under some Meridian, or East or West under the Equator. And in these Cases, every Degree requires an allowance of twenty Leagues; every twenty Leagues will make a Degree difference in the sailing: so that here needs no further Precept than the Rule of Proportion in the Chapter of Lines.

But if the Course hold East or West, or any of the Parallels to the Equator,

As the Radius,
is to twenty Leagues, the Measure of one Degree at the Equator:
So the Sine of the Complement of the Latitude,
to the Measure of Leagues answering to one Degree in that Latitude.

Where-

Gr.	m.	Leag.
0	0	20
18	12	19
25	15	18
31	48	17
36	52	16
41	25	15
45	34	14
49	28	13
53	8	12
56	38	11
60	0	10
63	15	9
66	25	8
69	30	7
72	32	6
75	31	5
78	28	4
81	23	3
84	15	2
87	8	1

Wherefore I take 20 Leagues out of the Line of Lines, and make it a parallel Radius, by fitting it over in the Sines of 90 and 90: ſo his parallel Sine taken out of the Complement of the Latitude, and meaſured in the Line of Lines, ſhall ſhew the number of Leagues required.

Thus in the Latitude of 18 *gr.* 12 *m.* we ſhall find 19 Leagues anſwering to one Degree of Longitude, and 18 Leagues in the Latitude of 25 *gr.* 15 *m.* as in this Table.

This may be done more readily without opening the Sector, by doubling the Sine of the Complement of the Latitude, as may appear in the ſame Example.

It may alſo be done by the Line of Meridians, either upon the Sector, or upon the Chart: For if we open a pair of Compaſſes to the quantity of one Degree of Longitude in the Equator, or one of his Parallels, and meaſure it in the Meridian line, ſetting one Foot as much above the Latitude given, as the other falleth beneath it, ſo that the Latitude may be in the middle between the Feet of the Compaſſes, the number of Leagues intercepted ſhall be that which was required.

But if the Courſe hold upon any of the *Rumbs*, between a Parallel of the Equator and the Meridian, we are to conſider (beſides the Equator of the World to which we tend, which muſt be always known),

1. The difference of Longitude, at leaſt in general.
2. The difference of Latitude, and that in particular.
3. The Rumb whereon the Courſe holds.
4. The diſtance upon the *Rumb*, which is the diſtance which we are here to conſider, and is always ſomewhat greater than the like diſtance upon a greater Circle. And for theſe, firſt, I ſhew in general this third Propoſition.

3. *To find how many Leagues do answer to one Degree of Latitude in every several Rumb.*

The Seamans Compass is commonly divided into 32 Points; the half, into 16; the quarter, into 8; which have their names of *N NbE, NNE, &c.* according to those parts of the World to which they point. Answerable to these Points, are the Rumbs upon their Chart; each quarter divided into 8, each Rumb 11 *gr.* 15 *m.* distant one from the other: The first Rumb being that which is 11 *gr.* 15 *m.* distant from the Meridian; the second, 22 *gr.* 30 *m.* the third, 33 *gr.* 45 *m.* and so the rest. And (if they have need of smaller parts) they subdivide each Rumb into quarters, allowing 2 *gr.* 48 *m.* to the first quarter, 5 *gr.* 37 to the half Rumb, *&c.* as in the Table following.

As the Sine of the Complement of the Rumb from the Meridian,
is to 20 Leagues, the Measure of one Degree of the Meridian:
So is the Radius,
to the Leagues answering to one Degree upon the Rumb.

As if in sailing *NE b N*, from 50 *gr.* of North Latitude, it were required how many Leagues the Ship should run before it could come to 51 *gr.* of Latitude, because this is the third Rumb, and the Inclination thereof 33 *gr.* 45 *m.* I would take 20 Leagues, *&c.*

Wherefore I take 20 Leagues out of the Line of Lines, and make it a parallel Sine of 56 *gr.* 15 *m.* the Complement of the Rumb from the Meridian; so his parallel Radius taken and measured in the Line of Lines, shall shew me 24 for the number of Leagues required.

Rumbs.	Inclinat. to the Merid. Gr.	M	Number of Leagues. Lgs.	Pa.
	2	49	20	02
	5	37	20	10
	8	26	20	22
1	11	15	20	39
	14	4	20	62
	16	52	20	90
	19	41	21	24
2	22	30	21	65
	25	19	22	12
	28	7	22	68
	30	56	23	32
3	33	45	24	05
	36	34	24	90
	39	22	25	87
	42	11	26	99
4	45	0	28	8
	47	49	29	78
	50	37	31	52
	53	26	33	57
5	56	15	36	00
	52	4	38	90
	61	52	42	43
	64	41	46	78
6	67	30	52	26
	70	19	59	37
	73	7	68	90
	75	56	82	31
7	78	45	102	52
	81	34	136	30
	84	22	205	24
	87	11	407	60
8	90	0	Infinita.	

And thus in the firſt Rumb from the Meridian we ſhall find 20 Leagues 39 parts anſwering to one Degree of Latitude, and 21 Leagues 65 parts in the ſecond Rumb, *&c.* as in this Table, where we ſubdivide each League into a hundred parts, and ſhew beſides what Inclination the Rumb hath to the Meridian.

This may be done more readily without opening the Sector, by doubling the Secant of the Rumb, as may appear in the ſame Example.

It may alſo be done upon the Chart, if firſt we draw the Rumb; then we take the diſtance upon the Rumb between two Parallels, and meaſure it in the Meridian-line, as far above the greater Latitude as beneath the leſſer. For the number of Leagues intercepted ſhall be that which was required.

For Example: In the ſecond Chart, *pag.* 113. I firſt draw the 8 Rumbs, from the Interſection of the Meridian with the Parallel of 50 *gr.* of Latitude, either by that which I have ſhewed before in the general Uſe of Sines, *Chap.* 11. *Prop.* 10. or by help of the Protraction laſt mentioned: For, laying the Center of the Protractor to the Point of Interſection (which is to be the Center of the Rumbs) and turning the Diameter of the Protractor until it be parallel to the Meridians of the Chart (which is then done, when the Meridians and Parallels in the Chart fall under like diviſions in the Protractor) I may make one prick at 11 *gr.* 15 *m.* another at 22 *gr.* 30 *m.* in the outward part of the Semicircle; and ſo the reſt.

Or, having neither Sector nor Protractor, I would have a Line of Chords ſet on the ſide of the Ruler which I am to uſe, from which I may take 60 *gr.* and with that extent ſetting one Foot of the Compaſſes in the former Point of Interſection, draw an occult Ark of a Circle, and therein prick down the former Arks from the Meridian, as in *Chap.* 11. *Prop.* 10. So theſe Arks being pricked down by either of theſe ways, the Right Lines drawn through the Center and thoſe pricks, ſhall be the Rumbs required.

The Rumbs being drawn, I take the diſtance between the Parallels of 50 and 51 *gr.* upon A C, the third Rumb; and

and meaſuring it in the Meridian-line, I find the Compaſſes to reach from above $\frac{1}{10}$ of a Degree below the Parallel of 50, but $\frac{1}{10}$ above the Parallel of 51 *gr.* intercepting 1 *gr.* $\frac{2}{10}$ or 24 Leagues, ſuch as 20 make a degree.

Again, I take the diſtance upon the ſame Rumb between the Parallel of 54 and 55 *gr.* which I find to be ſomewhat longer than the former diſtance between the Parallels of 50 and 51; but meaſuring it in the Meridian Line, according to the Latitude of the Parallel, I find but 1 *gr.* $\frac{2}{10}$ (or 24 Leagues) as before, for the number of Leagues anſwering to 1 Degree of Latitude upon this third Rumb.

And by the ſame reaſon, I may find the number of Leagues anſwering to a Degree of Latitude upon the reſt of the Rumbs agreeable to the Table.

This conſidered in general, I ſhew more particularly in twelve Propoſitions following, how of theſe four any two being given, the other two may be found, both by *Mercator's* Chart, and by this *Sector*.

1. *By one Latitude, Rumb, and Diſtance, to find the Difference of Latitudes.*

As the Radius,
to the Sine of the Complement of the Rumb from the Meridian:
So the Diſtance upon the Rumb,
to the difference of Latitudes.

Let the Place given be A, in the Latitude of 50 *gr.* C in a greater Latitude, but unknown, the diſtance upon the Rumb being 6 *gr.* between them, and the Rumb the third from the Meridian.

Firſt, I take 6 *gr.* from the diſtance upon the Rumb, out of the Line of Lines, and make it a Parallel Radius, by putting it over in the Sines of 90 and 90: Then keeping the Sector at this Angle, I take out the Parallel Sine of 56 *gr.* 15 *m.* which is the Sine of the Complement of the third Rumb from the Meridian, and meaſuring it in the Line of Lines, I find it to be 5 *gr.* and ſuch is the difference of Latitude required.

Or, I may take out the Sine of 56 *gr.* 15 *m.* for the Complement of the third Rumb from the Meridian, and make it a parallel Radius; then keeping the Sector at this Angle, I take 6 *gr.* for the distance, either out of the Line of Lines, or any other Scale of Equal parts, or else out of the Meridian Line, and lay it on both sides of the Sector from the Center, either on the Line of Lines or Sines: so the Parallel taken from the Terms of this distance, and measured in the same Scale wherein the distance was measured, shall shew the difference of Latitude to be 5 *gr.* as before.

But in shorter distances, such as fall within the compass of a days sailing, this Work will hold much better; as may appear by comparing the Work with the Table following, where the Numbers in the front do signifie the Leagues; those in the side, the Rumb; and the rest in the middle, the difference of Latitude.

In the Chart let a Meridian A B be drawn through A, and in A with A B make an Angle of the Rumb B A C: Then open the Compasses, according to the Latitude of the Places, to E F the quantity of 6 *gr.* in the Meridian, transferring them into the Rumb from A to C, and through C draw the Parallel B C, crossing the Meridian A B in B: so the Degrees in the Meridian from A to B shall shew the difference of Latitude to be 5 *gr.*

2. *By the Rumb and both Latitudes to find the Distance upon the Rumb.*

As the Sine of the Complement of the Rumb from the Meridian, is to the Radius:
So is the Difference of Latitudes,
to the Distance upon the Rumb.

As if the Places given were A in the Latitude of 50 *gr.* C in the Latitude of 55 *gr.* and the Rumb the third from the Meridian.

Here I may take 5 *gr.* for the difference of Latitude out of the Line of Lines, and put it over in the Sine of 56 *gr.* 15 *m.* for the Com-

A Table of Leagues, Rumbs, and Difference of Latitude.

Le	100	80	60	40	20	19	18	17	16	15
Rum.	G.M.	G.M.	G.M.	G.M	M	M	M	M	M	M
	5 0	4 0	3 0	2 0	60	57	54	51	48	45
	4 59	3 59	2 59	1 59	60	57	54	51	48	45
	4 58	3 58	2 59	1 59	60	57	54	51	48	45
	4 56	3 58	2 58	1 58	59	56	53	50	47	44
1	4 54	3 57	2 50	1 57	59	56	53	50	47	44
	4 51	3 53	2 55	1 56	58	56	52	50	47	43
	4 47	3 50	2 52	1 55	57	55	52	49	46	43
	4 42	3 46	2 49	1 53	56	54	51	48	45	42
2	4 37	3 42	2 46	1 51	55	53	50	47	44	41
	4 31	3 37	2 43	1 48	54	52	49	40	42	40
	4 25	3 32	2 39	1 46	53	50	48	45	42	39
	4 17	3 26	2 34	1 43	51	49	46	44	41	38
3	4 10	3 20	2 30	1 40	50	47	45	42	40	37
	4 1	3 13	2 25	1 36	48	46	43	41	39	36
	3 52	3 5	2 19	1 32	46	44	42	39	37	35
	3 42	2 58	2 13	1 28	44	42	40	38	36	33
4	3 32	2 50	2 7	1 25	42	40	38	36	34	32
	3 22	2 41	2 1	1 21	40	38	36	34	32	30
	3 10	2 32	1 54	1 16	38	36	34	32	30	28
	2 59	2 23	1 47	1 12	36	34	32	30	29	27
5	2 47	2 14	1 40	1 7	33	32	30	28	27	25
	2 34	2 3	1 32	1 2	31	29	28	26	25	23
	2 22	1 53	1 25	0 57	28	27	25	24	23	22
	2 8	1 40	1 17	0 52	26	24	23	22	21	19
6	1 55	1 32	1 8	0 46	23	22	21	20	18	17
	1 41	1 20	1 0	0 40	20	19	18	17	16	15
	1 27	1 9	0 52	0 35	17	16	16	15	14	13
	1 13	0 58	0 44	0 30	15	14	13	12	12	11
7	0 59	0 47	0 35	0 24	12	11	11	10	9	9
	0 44	0 36	0 16	0 18	9	8	8	7	7	7
	0 30	0 24	0 18	0 12	6	6	5	5	5	4
	0 15	0 12	0 9	0 9	3	3	3	3	2	2
8	0 0	0 0	0 0	0 0	0	0	0	0	0	

A Table of Leagues, Rumbs, and Difference of Latitudes.

14	13	12	11	10	9	8	7	6	5	4	3	2	1	Le
M.	M	M	M	M	M.	M	M	M	M	M	M	M	M	Rum.
40	39	36	33	30	27	24	21	18	15	12	9	6	3	
42	39	36	33	30	27	24	21	18	15	12	9	6	3	
42	39	36	33	29	27	24	21	18	15	12	9	6	3	
41	38	35	32	29	26	24	21	18	15	12	9	6	3	1
41	38	35	32	29	26	23	20	17	14	12	9	6	3	
40	37	34	32	29	26	23	20	17	14	11	9	6	3	
40	37	34	31	28	25	23	20	17	14	11	8	6	3	
39	36	33	31	28	25	22	19	17	14	11	8	6	3	2
38	35	33	30	27	24	22	19	16	14	11	8	5	3	
37	34	32	29	26	24	21	19	16	14	11	8	5	3	
36	33	31	28	26	23	21	18	15	13	10	8	5	3	
35	32	30	27	25	22	20	17	15	12	10	7	5	2	3
34	30	29	26	24	22	19	17	14	12	10	7	5	2	
33	30	28	25	23	21	19	16	14	12	9	7	5	2	
31	29	27	24	22	20	18	16	13	11	9	7	4	2	
30	28	25	23	21	19	17	15	13	11	8	6	4	2	4
28	26	24	22	20	18	16	14	12	10	8	6	4	2	
27	25	23	21	19	17	15	13	11	10	8	6	4	2	
25	23	21	20	18	16	14	13	11	9	7	5	4	2	
23	22	20	18	17	15	13	12	10	8	7	5	3	2	5
22	20	18	17	15	14	12	11	9	8	6	5	3	2	
20	18	17	16	14	13	11	10	8	7	6	4	3	1	
18	17	15	14	13	12	10	9	8	6	5	4	3	1	
16	15	14	13	11	10	9	8	7	6	5	3	2	1	6
14	13	12	11	10	9	8	7	6	5	4	3	2	1	
12	11	10	10	9	8	7	6	5	4	3	3	2	1	
10	9	9	8	7	7	6	5	4	4	3	2	1	1	
8	8	7	6	6	6	5	4	3	3	2	2	1	1	7
6	6	5	5	4	5	4	3	3	2	2	1	1	1	
4	4	4	3	3	3	2	2	2	1	1	1	1	0	
2	2	2	2	1	1	1	1	1	1	1	1	1	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	8

Complement of the third Rumb from the Meridian. Then keeping the Sector at this Angle, I take out the Parallel Radius, and meaſuring it in the Line of *Lines*; I find it to be 6 *gr.* and ſuch is the diſtance upon the Rumb, which was required.

Or I may take the Lateral Radius, and make it a Parallel Sine of 56 *gr.* 15 *m.* the Complement of the Rumb from the Meridian: then keeping the Sector at this Angle, I take 5 *gr.* for the difference of Latitude, either out of the Line of *Lines*, or out of ſome other Scale of equal parts, and lay it on both ſides of the Sector from the Center, either on the Line of *Lines* or of *Sines*: ſo the Parallel taken from the terms of his difference, and meaſured in the ſame Scale with the the difference, ſhall ſhew the diſtance upon the Rumb to be 6 *gr.* or 120 Leagues.

Or keeping the *Sector* at this Angle, I may take the difference between 50 *gr.* and 55 *gr.* out of the Meridian Line, and meaſuring it in the Equator, I ſhall find it to be equal to 8 *gr.* 22 *p.* of the Equator. Wherefore I take the Parallel between 822 and 822 out of the Line of *Lines*, and meaſuring it in the Line of *Lines*, I ſhall find it to be 989; which ſhews that according to this projection, the diſtance upon this third Rumb, anſwerable to the former difference of Latitudes, will be equal to 9 *gr.* 89 *p.* of the Equator.

Or the Sector remaining at this Angle, I may take the difference between 50 *gr.* and 55 *gr.* out of the Meridian Line, and lay it from the Center on both ſides of the Sector, either on the Line of Lines or of Sines: ſo the Parallel taken from the terms of this difference, ſhall be the very Line of diſtance required, the ſame with A C or E F upon the Chart; which may ſerve for the better pricking down of the diſtance upon the Rumb, without taking it forth of the Meridian Line, as in the former *Propoſition*.

Or if the Rumb fall nearer to the Equator, that the lateral Radius cannot be fitted over in it, this Propoſition may be wrought by Parallel entrance.

For, if I firſt take out the Sine of 56 *gr.* 15 *m.* and make it a parallel Radius, by fitting it over in the Sines of 90 and 90, or in the ends of the Line of *Lines*, and then take 5 *gr.* for the difference of Latitudes out of the Line of *Lines*, and carry it parallel to the former, I ſhall find it to croſs both Lines of *Lines* in the Points of 6: and ſo it gives the ſame diſtance as before.

Or if the diſtance be ſmall, it may be found by the former Table.

For,

For the Rumb being found in the side of the Table, and the difference of Latitude in the same Line; the top of the Column wherein the difference of Latitude was found, shall give the number of Leagues in the distance required.

Or we may find this distance in the Table of Rumbs in the fifth *Proposition* following. For according to the example, look into the Table of the third Rumb for 5 *gr.* of Latitude, and there we shall find 6 *gr.* 10 parts under the title of distance.

So if the difference of Latitude upon the same Rumb were 50 *gr.* the distance would be 60 *gr.* 13 parts. If the difference of Latitude upon the same Rumb were only ½ of a degree, the distance would be only 60 parts, such as 100 do make a degree.

In the Chart let a Meridian A B be drawn through A, and Parallels of Latitude through A and C; and then in A, with A B, make an Angle of the Rumb B A C: so the distance take from A to C, and measured in the Meridian Line, according to the Latitude of the places, shall be found to be 6 *gr.* or 120 Leagues. And such is the distance required.

3. *By the distance and both Latitudes, to find the Rumb.*

As the distance upon the Rumb,
to the difference of Latitudes:
So is the Radius,
to the Sine of the Complement of the Rumb from the Meridian.

As if the places given were A, in the Latitude of 50 *gr.* C in the Latitude of 55 *gr.* the distance between them being 6 *gr.* upon the Rumb. First I take 6 *gr.* for the distance upon the Rumb, and lay it on both sides of the *Sector* from the Center; then out of the same Scale I take 5 *gr.* for the difference of Latitude, and to it open the *Sector* in the terms of the former distance: so the parallel Radius taken and measured in the Sines, doth give 56 *gr.* 15 *m.* the Complement whereof 33 *gr.* 45 *m.* is the Angle of the Rumbs inclination to the Meridian, which was required.

In the Chart let a Meridian A B be drawn through A, and Parallels of Latitude, both through A and C; then open the Compasses according to the Latitude of the places to E F, the quantity of 6 *gr.* in the Meridian, and setting one foot in A, turn the other till it cross the Parallel

Parallel B C in C, and draw the right Line A C: ſo the Angle B A C ſhall ſhew the inclination of the Rumb to the Meridian to be 33 *gr.* 45 *m.* as before.

Theſe three laſt Propoſitions depend one on the other, and may be wrought as truly by the Common Sea-Chart as by this of *Mercators* Projection: and therefore in working them by the *Sector*, the diſtance and the difference of Latitudes may as well or better be taken out of the Line of *Lines* (which here repreſenteth the Equator) or any other Line of equal parts, as out of the inlarged degrees in the Meridian Line. But in the Propoſitions following, the difference of Longitude muſt be taken out of the Equator; the difference of Latitudes and diſtance upon the Rumb muſt alwaies be taken out of the Meridian Line: which I therefore call the proper difference, and proper diſtance.

4. *By the Longitude and Latitude of two places to find the Rumb.*

As if the places given were A, in the Latitude of 50 *gr.* C in the Latitude of 55 *gr.* and the difference of Longitude between them were 5 *gr.* 30 *m.*

In the Chart let Meridians and Parallels be drawn through A and C, and a ſtraight Line for the Rumb from A to C; then by that we ſhewed *Cap.* 2. *Propoſition* 9. inquire the quantity of the Angle B A C, and it ſhall be found to be 33 *gr.* 45 *m.* which is the third Rumb from the Meridian. Wherefore the proportion holds for the Sector,

As A B the proper difference of Latitude:
is to B C the difference of Longitude:
So is A B Radius,
to B C, the Tangent of the Rumb from the Meridian.

According to this I take the proper difference of Latitude from 50 *gr.* to 55 *gr.* out of the Line of Meridians, and lay it on both ſides of the *Sector* from the Center; then I take the difference of Longitude 5 *gr.* ½ out of the Line of *Lines*, and to it open the Sector in the terms of the former difference of Latitudes, ſo the Parallel Radius taken from between 90 and 90: and meaſured in the greater

Tangent on the ſide of the *Sector*, doth give 33 *gr.* 45 *m.* for the Rumb required.

But if the Rumb fall nearer to the Equator;

As A D the difference of Longitudes,
is to D C the proper difference of Latitudes:
So A D the Radius,
to D C the Tangent of the Rumb from the Equator.

According to this I take the former difference of Latitudes from 50 *gr.* to 55 *gr.* out of the Line of Meridians, and to it open the *Sector* in the terms of the difference of Longitude reckoned in the Line of *Lines* from the Center, ſo the Parallel Radius taken and meaſured in the Tangent, doth give 56 *gr.* 15 *m.* for the Rumb from the Equator: which is the Complement to the former 33 *gr.* 45 *m.* and ſo both ways it is found to be the third Rumb from the Meridian.

But if this Rumb were to be found in the common Sea-chart, it ſhould ſeem to be above 47 *gr.* which is more than the fourth Rumb from the Meridian.

5. *By the Rumb and both Latitudes, to find the difference of Longitude.*

As if the places given were A, in the Latitude of 50 *gr.* and C in the Latitude of 55 *gr.* and the Rumb the third from the Meridian.

In the Chart, let a Meridian be drawn through A, and a Parallel of Latitude through C, then in A, with A B, make the Angle of the Rumb from the Meridian B A C, (as was ſhewed *Cap.* 2. *Prop.* 10.) So the degrees in the Parallel between B and C, ſhall be found to be 5 *gr.* $\frac{1}{3}$; the difference of Longitude which was required. Wherefore the proportion holds for the *Sector*,

As A B the Radius,
to B C the Tangent of the Rumb from the Meridian:
So A B the proper difference of the Latitudes,
to B C the difference of Longitude.

Accord-

According to this we may take the Tangent of the Rumb which is here 33 *gr.* 45 *m.* from the Meridian, out of the greater Tangent on the ſide of the *Sector*, and putting it over between 90 and 90, make it a Radius: then keeping the *Sector* at this Angle, take the proper difference of Latitudes from 50 *gr.* to 55 *gr.* out of the Line of Meridians, and lay it on both ſides of the Sector from the Center: ſo the Parallel taken from the terms of this difference, and meaſured in the Line of *Lines*, ſhall ſhew the difference of Longitude to be 5 *gr.* ½.

As D C the Tangent of the Rumb from the Equator,
to A D the Radius:
So C D the proper difference of the Latitudes,
to A D the difference of Longitude.

According to this, we may beſt work by Parallel entrance, firſt taking 56 *gr.* 15 *m.* for the Angle of the Rumb from the Equator, out of the greater Tangent, and make it a Parallel Radius: then take the proper difference of Latitudes out of the Line of Meridians, and carry it Parallel to the former: ſo we ſhall find it to croſs the Line of Lines in 5 *gr.* ½. And this is the difference of Longitude required, the ſame as before.

But if this difference were to be found by the common Sea-chart, it ſhould ſeem to be only 3 *gr.* 20 *m*, which is more than two degrees leſs than the truth. And yet this error would be greater, if either the Latitude be greater, or the Rumb fall nearer the Equator, as may appear by comparing the common Sea-chart with the Table following.

The first Rumb, from the Meridian.	North and by East, South and by East,	North and by West. South and by West.

La	Long	Dist.	La	Long.	Dist.	La	Long.	Dist.
Gr	Gr. Par	Gr. Par.	Gr	Gr. Par	Gr. Par	Gr	Gr. Par.	Gr. Par.
0	0	0	30	6 20	30 59	00	15 10	61 18
1	20	1 02	31	6 49	31 61	61	15 41	62 20
2	40	2 04	32	6 72	32 63	62	15 83	63 21
3	60	3 06	33	6 96	33 65	63	16 26	64 23
4	80	4 08	34	7 20	34 67	64	16 71	65 25
5	1 00	5 10	35	7 44	35 69	65	17 17	66 27
6	1 20	6 12	36	7 68	36 71	66	17 65	67 29
7	1 40	7 14	37	7 92	37 73	67	18 15	68 31
8	1 60	8 16	38	8 17	38 75	68	18 77	69 33
9	1 80	9 18	39	8 43	39 77	69	19 21	70 35
10	2 00	10 20	40	8 70	40 78	70	19 78	71 37
11	2 20	11 22	41	8 96	41 80	71	20 35	72 36
12	2 40	12 24	42	9 22	42 82	72	21 07	73 41
13	2 61	13 25	43	9 50	43 84	73	21 60	74 43
14	2 81	14 27	44	9 76	44 86	74	22 36	75 45
15	3 02	15 29	45	10 04	45 88	75	23 16	76 47
16	3 22	16 31	46	10 33	46 90	76	23 90	77 49
17	3 43	17 33	47	10 62	47 92	77	24 75	78 51
18	3 64	18 35	48	10 91	48 94	78	25 67	79 53
19	3 85	19 37	49	11 21	49 96	79	26 67	80 55
20	4 06	20 39	50	11 52	50 98	80	27 76	81 57
21	4 27	21 41	51	11 83	52 0	81	28 97	82 59
22	4 49	22 43	52	12 15	53 2	82	30 32	83 61
23	4 70	23 45	53	12 47	54 4	83	31 84	84 63
24	4 92	24 47	54	12 81	55 6	84	33 61	85 65
25	5 14	25 49	55	13 16	56 8	85	35 69	86 67
26	5 36	26 51	56	13 50	57 10	86	38 24	87 69
27	5 58	27 53	57	13 86	58 12	87	41 52	88 71
28	5 80	28 55	58	14 23	59 14	88	46 15	89 73
29	6 03	29 57	59	14 62	60 16	89	54 06	90 75
30	6 26	30 59	60	15 01	61 18	90		

The second Rumb from the Meridian.			North, North-east, South, South-east.			North, North-west, South, South-west.		
La	Long.	Dist.	La	Long.	Dist.	La	Long.	Dist.
Gr	Gr. Par.	Gr. Par.	Gr	Gr. Par.	Gr. Par.	Gr	Gr. Par.	Gr. Par.
0	0	0	30	13 03	32 47	60	31 29	64 94
1	0 42	1 08	31	13 51	33 54	61	32 09	66 03
2	0 83	2 16	32	14 00	34 54	62	32 96	67 11
3	1 24	3 25	33	14 49	35 72	63	33 86	68 19
4	1 65	4 33	34	15 00	36 80	64	34 79	69 27
5	2 07	5 41	35	15 50	37 88	65	35 75	70 35
6	2 49	6 49	36	16 00	38 97	66	36 75	71 44
7	2 91	7 57	37	16 51	40 05	67	37 80	72 52
8	3 32	8 66	38	17 03	41 13	68	38 88	73 60
9	3 74	9 74	39	17 56	42 21	69	40 00	74 68
10	4 16	10 82	40	18 10	43 30	70	41 19	75 77
11	4 59	11 90	41	18 65	44 38	71	42 43	76 85
12	5 01	12 99	42	19 20	45 46	72	43 74	77 99
13	5 43	14 07	43	19 76	46 54	73	45 11	79 01
14	5 85	15 15	44	20 33	47 62	74	46 57	80 10
15	6 28	16 23	45	20 92	48 71	75	48 12	81 18
16	6 71	17 32	46	21 50	49 79	76	49 78	82 26
17	7 14	18 40	47	22 11	50 87	77	51 55	83 34
18	7 58	19 48	48	22 72	52 95	78	53 46	84 42
19	8 91	20 56	49	23 35	53 03	79	55 54	85 52
20	8 45	21 65	50	23 98	54 12	80	57 82	86 59
21	8 90	22 73	51	24 63	55 20	81	60 33	87 67
22	9 34	23 81	52	25 30	56 28	82	63 13	88 76
23	9 79	24 89	53	25 98	57 37	83	66 32	89 84
24	10 24	25 98	54	26 69	58 45	84	69 99	90 92
25	10 70	27 06	55	27 39	59 53	85	74 32	92 00
26	11 16	28 14	56	28 12	60 61	86	79 63	93 09
27	11 62	29 22	57	28 87	61 79	87	86 46	94 17
28	12 08	30 31	58	29 64	62 78	88	96 10	95 25
29	12 55	31 39	59	30 44	63 86	89	112 57	96 38
30	13 03	32 47	60	31 25	64 94	90		

The third Rumb, from the Meridian.			North-east by North, South-east by South,			North-west by North, South-west by South,		
La Gr	Long Gr. Par	Dist. Gr. Par.	La Gr	Long. Gr. Par	Dist. Gr. Par	La Gr	Long. Gr. Par.	Dist. Gr. Par.
0	0	0	30	21 03	36 08	00	50 42	72 16
1	0 66	1 20	31	21 80	37 28	61	51 78	73 36
2	1 33	2 40	32	22 58	38 49	62	53 18	74 56
3	2 00	3 61	33	23 38	39 69	63	54 63	75 77
4	2 67	4 81	34	24 18	40 89	64	56 12	76 97
5	3 34	6 01	35	25 00	42 09	65	57 68	78 17
6	4 01	7 22	36	25 82	43 30	66	59 29	79 37
7	4 68	8 42	37	26 64	44 50	67	69 99	80 58
8	5 36	9 62	38	27 48	45 70	68	62 71	81 78
9	6 03	10 82	39	28 39	46 90	69	64 53	82 98
10	6 71	12 03	40	29 21	48 11	70	66 44	84 19
11	7 39	13 23	41	30 09	49 31	71	68 45	85 39
12	8 07	14 43	42	30 98	50 51	72	70 55	86 59
13	8 76	15 64	43	31 88	51 71	73	72 77	87 79
14	9 44	16 84	44	32 80	52 92	74	75 12	89 00
15	10 13	18 04	45	33 74	54 12	75	77 63	90 20
16	11 83	19 24	46	34 69	55 32	76	80 30	91 40
17	12 53	20 45	47	35 67	56 52	77	83 15	92 61
18	12 23	21 65	48	36 66	57 73	78	86 25	93 81
19	12 93	22 85	49	37 67	58 93	79	89 60	95 01
20	13 64	24 05	50	38 69	60 13	80	93 27	96 22
21	14 35	25 26	51	39 74	61 33	81	97 32	97 42
22	15 07	26 46	52	40 82	62 54	82	101 85	98 62
23	15 80	27 66	53	41 91	63 74	83	106 97	96 82
24	16 53	28 86	54	43 03	64 94	84	112 90	101 03
25	17 26	30 07	55	44 19	66 15	85	119 90	102 23
26	18 00	31 27	56	45 37	67 45	86	128 45	103 43
27	18 75	32 47	57	46 58	68 55	87	139 47	104 64
28	19 50	33 67	58	47 82	69 75	88	155 00	105 84
29	20 28	34 88	59	49 11	70 96	89	181 58	107 04
30	21 03	36 08	60	50 42	72 16	90		

The fourth Rumb, from the Meridian. *North-east, South east,* *North-west, South-west.*

La. Gr	Long. Gr. Par.	Dist. Gr. Par.	L. Gr	Long. Gr. Par.	Dist. Gr. Par.	La. Gr	Long. Gr. Par.	Dist. Gr. Par.
0	0	0	30	31 47	42 42	60	75 46	84 85
1	1 00	1 41	31	32 63	43 84	61	77 49	85 27
2	2 00	2 83	32	33 81	45 25	62	79 58	87 68
3	3 00	4 24	33	34 99	46 67	63	81 75	89 09
4	4 00	5 66	34	36 19	48 07	64	83 99	90 51
5	5 01	7 07	35	37 41	49 50	65	86 31	91 92
6	6 01	8 49	36	38 63	50 91	66	88 73	93 34
7	7 02	9 90	37	39 88	52 33	67	91 23	94 75
8	8 03	11 31	38	41 14	53 74	68	93 85	96 17
9	9 04	12 73	39	42 42	55 15	69	96 58	97 58
10	10 05	14 14	40	43 71	56 65	70	99 43	98 99
11	11 07	15 56	41	45 03	57 98	71	102 43	100 41
12	12 09	16 97	42	46 36	59 40	72	105 58	101 82
13	13 11	18 38	43	47 72	60 81	73	108 91	103 24
14	14 14	19 80	44	49 10	62 22	74	112 43	104 65
15	15 17	21 21	45	50 50	63 64	75	116 17	106 06
16	16 21	22 63	46	51 93	65 05	76	120 17	107 48
17	17 35	24 04	47	53 38	66 46	77	124 45	108 89
18	18 30	25 45	48	54 86	67 88	78	129 08	110 31
19	19 36	26 87	49	56 37	69 29	79	134 10	111 72
20	20 42	28 28	50	57 91	70 71	80	139 59	113 14
21	21 49	29 70	51	59 48	72 12	81	145 65	114 55
22	22 56	31 11	52	61 09	73 54	82	152 42	115 96
23	23 64	32 54	53	62 73	74 95	83	160 10	117 38
24	24 73	33 94	54	64 41	76 37	84	168 9	118 79
25	25 83	35 35	55	66 13	77 78	85	179 41	120 21
26	26 94	36 77	56	67 90	79 20	86	192 21	121 62
27	28 06	38 18	57	69 71	80 61	87	208 71	123 04
28	29 18	39 60	58	71 57	82 02	88	231 95	124 45
29	30 32	41 01	59	73 49	83 44	89	271 71	125 86
30	31 47	42 43	60	75 49	84 85	90		

The fifth Rumb from the Meridian. *North east and by East.* *South-east and by East.* *North west and by West.* *South-west and by West.*

La Gr	Long. Gr. Par.	Dist. Gr. Par.	La Gr	Long. Gr. Par.	Dist. Gr. Par.	La Gr	Long. Gr. Par.	Dist. Gr. Par.
0	0	0	30	47 10	54 00	60	112 9	108 01
1	1 49	1 80	31	48 84	55 80	61	115 97	109 80
2	2 99	3 60	3	50 60	57 60	62	119 10	111 60
3	4 49	5 40	33	52 37	59 40	6	122 34	113 40
4	6 00	7 20	34	54 16	61 20	64	125 70	115 20
5	7 50	9 00	35	55 98	63 00	65	129 18	117 00
6	9 00	10 80	36	57 82	64 80	66	132 78	118 80
7	10 50	12 60	37	59 68	66 60	67	136 54	120 60
8	12 01	14 40	38	61 57	68 40	63	140 45	122 40
9	13 52	16 20	39	63 48	70 20	69	144 53	124 20
10	15 04	18 00	40	65 42	72 00	70	148 81	126 00
11	16 56	19 80	41	66 39	73 80	71	153 30	127 80
12	18 09	21 60	42	69 39	75 60	72	158 00	129 60
13	19 62	23 40	43	61 43	77 40	73	163 00	131 40
14	21 16	25 20	44	73 48	79 20	74	168 26	133 20
15	22 70	27 00	45	75 58	81 00	75	173 86	135 00
16	24 26	28 80	46	77 72	82 80	76	179 84	136 80
17	25 82	30 60	47	79 89	84 60	77	186 26	138 60
18	27 39	32 40	48	82 10	86 40	78	193 17	140 40
19	28 97	34 20	49	84 36	88 10	79	200 69	142 20
20	30 55	36 00	50	86 67	90 00	80	208 91	144 00
21	32 15	37 80	51	89 03	91 80	81	217 98	145 80
22	33 76	39 60	52	91 43	93 60	82	228 13	147 60
23	35 38	41 40	53	92 88	95 40	83	239 61	149 40
24	37 01	43 20	54	96 40	97 20	84	252 85	151 20
25	38 66	45 00	55	98 98	99 00	85	268 51	153 00
26	40 32	46 80	56	101 62	100 80	86	287 67	154 80
27	42 00	48 60	57	104 33	102 60	87	312 36	156 60
28	43 67	50 40	58	107 12	104 40	88	345 15	158 40
29	45 38	52 20	59	109 98	106 20	89	406 72	160 20
30	47 10	54 00	60	112 93	108 00	90		

The sixth Rumb from the Meridian. *East North east, West North west,* *East South-east. West South-west.*

La Gr	Long. Gr. Par.	Dist. Gr. Par.
0	0	0
1	2 41	2 61
2	4 83	5 23
3	7 25	7 84
4	9 60	10 45
5	12 08	13 06
6	14 51	15 68
7	16 94	18 29
8	19 37	20 90
9	21 81	23 52
10	24 26	26 13
11	26 71	28 74
12	29 17	31 36
13	31 65	33 97
14	34 14	36 58
15	36 63	39 20
16	39 13	41 81
17	41 65	44 42
18	44 18	47 03
19	46 75	49 65
20	49 29	52 26
21	51 87	54 87
22	54 47	57 49
23	57 08	60 10
24	59 71	62 71
25	62 36	65 33
26	65 04	67 94
27	67 74	70 55
28	70 46	73 17
29	73 20	65 78
30	75 98	78 39

La Gr	Long. Gr. Par.	Dist. Gr. Par.
30	75 98	78 39
31	78 78	81 00
32	81 61	83 62
33	84 48	86 23
34	87 37	88 84
35	90 30	91 46
36	93 27	94 07
37	96 27	96 68
38	99 31	99 30
39	102 40	101 91
40	105 53	104 52
41	108 71	107 14
42	111 93	109 75
43	115 20	112 36
44	118 53	114 97
45	121 92	117 59
46	125 36	120 20
47	128 87	122 81
48	132 44	125 43
49	136 09	128 04
50	139 81	130 65
51	143 60	133 27
52	147 47	135 88
53	151 44	138 46
54	155 50	141 10
55	159 66	143 72
56	163 93	146 33
57	168 31	148 95
58	172 80	151 56
59	177 42	154 17
60	182 18	156 78

La Gr	Long. Gr. Par.	Dist. Gr. Par.
00	182 18	156 78
61	187 0,	159 40
62	192 13	162 01
63	197 36	164 62
64	202 77	167 14
65	208 38	169 85
66	214 20	172 46
67	220 25	175 08
68	226 57	177 69
69	233 15	180 30
70	240 06	182 92
71	247 37	185 53
72	254 90	188 14
73	262 91	190 75
74	271 43	193 37
75	280 46	195 98
76	290 11	198 59
77	300 46	201 21
78	311 62	203 82
79	323 73	206 43
80	337 00	209 05
81	351 64	211 66
82	368 00	214 27
83	386 51	216 89
84	407 89	219 50
85	433 13	222 11
86	464 05	224 73
87	503 88	227 34
88	560 00	229 95
89	656 08	232 56
90		

The seventh Rumb from the Meridian. East and by North, West and by North, East and by South, West and by South.

Lat	Long.	Dist.
Gr	Gr. Par.	Gr. Par.
0	0	0
1	05 02	5 12
2	10 05	10 25
3	15 08	15 38
4	20 12	20 50
5	25 16	25 63
6	30 21	30 75
7	35 27	35 88
8	40 34	41 00
9	45 42	46 13
10	50 52	51 26
11	55 63	56 38
12	60 77	61 51
13	65 92	66 63
14	71 09	71 76
15	76 28	76 88
16	81 50	82 01
17	86 75	87 14
18	92 02	92 26
19	97 31	97 39
20	102 64	102 51
21	108 01	107 64
22	113 42	112 77
23	118 87	117 89
24	124 35	123 02
25	129 87	128 14
26	135 44	133 27
27	141 05	138 40
28	146 71	143 52
29	152 44	148 65
30	158 23	158 23

Lat	Long.	Dist.
Gr	Gr. Par.	Gr. Par.
30	158 23	153 77
31	64 00	158 90
32	169 96	164 02
33	175 92	169 15
34	181 95	174 28
35	188 04	179 40
36	194 22	184 53
37	200 48	189 65
38	206 82	194 78
39	213 24	199 90
40	219 76	205 03
41	226 37	210 16
42	233 08	215 28
43	239 90	220 41
44	246 84	225 53
45	253 89	230 66
46	261 05	235 79
47	268 36	240 91
48	275 80	246 04
49	283 40	251 16
50	291 13	256 29
51	299 03	261 41
52	307 11	266 54
53	315 37	271 66
54	323 82	276 79
55	332 48	281 92
56	341 36	287 04
57	350 47	292 17
58	359 81	297 30
59	369 45	302 43
60	379 35	307 55

Lat	Long.	Dist.
Gr	Gr. Par.	Gr. Par.
60	379 35	307 55
61	389 56	312 67
62	400 10	317 80
63	410 98	322 93
64	422 26	328 05
65	433 94	333 18
66	446 03	338 30
67	458 66	343 43
68	471 80	348 55
69	485 52	353 68
70	499 89	358 81
71	514 94	363 93
72	530 79	369 06
73	547 52	374 18
74	565 22	379 31
75	584 03	384 43
76	604 13	389 56
77	625 67	394 69
78	648 91	399 81
79	674 15	404 94
80	701 75	410 06
81	732 25	415 19
82	766 30	420 32
83	804 86	425 44
84	849 38	430 57
85	901 98	435 69
86	966 31	440 82
87	1049 26	445 94
88	1166 11	451 07
89	1366 23	456 20
90		

The eighth Rumb of East and West, with the Longitude answering to one deg. of distance, and the distance belonging to one degree of Longitude.

La Gr	Long. Gr. Par.	Dist. Parts.	La Gr	Long. Gr. Par.	Dist. Parts.	La Gr	Long. Gr. Par.	Dist. Parts.
0	0	10000	30	1 25	86 60	00	2 00	50 00
1	1 00	99 98	31	1 17	85 71	61	2 00	48 48
2	1 00	99 94	32	1 18	84 80	62	2 13	46 94
3	1 00	99 86	33	1 19	83 86	63	2 20	45 40
4	1 00	99 75	34	1 21	82 90	64	2 28	43 83
5	1 00	99 62	35	1 22	81 91	65	2 37	42 26
6	1 01	99 45	36	1 24	80 90	66	2 46	40 67
7	1 01	99 25	37	1 25	79 86	67	2 56	39 07
8	1 01	99 02	38	1 27	78 80	68	2 67	37 46
9	1 01	98 76	39	1 29	77 71	69	2 79	35 83
10	1 02	98 48	40	1 31	76 60	70	2 92	34 20
11	1 02	98 16	41	1 33	75 47	71	3 07	32 55
12	1 02	97 81	42	1 35	74 31	72	3 24	30 90
13	1 03	97 43	43	1 37	73 13	73	3 42	29 23
14	1 03	97 03	44	1 39	71 93	74	3 63	27 56
15	1 03	96 59	45	1 41	70 71	75	3 86	25 88
16	1 04	96 12	46	1 44	69 46	76	4 13	24 19
17	1 04	95 63	47	1 47	68 20	77	4 44	22 49
18	1 05	95 10	48	1 49	66 91	78	4 81	20 78
19	1 06	94 55	49	1 52	65 60	79	5 24	19 08
20	1 06	93 97	50	1 55	64 28	80	5 76	17 36
21	1 07	93 35	51	1 59	62 63	81	6 39	15 64
22	1 08	92 72	52	1 62	61 56	82	7 18	13 91
23	1 09	92 05	53	1 66	60 18	83	8 20	12 18
24	1 09	91 35	54	1 70	58 77	84	9 57	10 45
25	1 10	90 63	55	1 74	57 35	85	11 47	8 71
26	1 11	89 88	56	1 79	55 92	86	14 33	6 97
27	1 12	89 10	57	1 84	54 46	87	19 11	5 23
28	1 13	88 29	58	1 89	52 99	88	28 65	3 49
29	1 14	87 46	59	1 94	51 50	89	57 30	1 74
30	1 15	86 60	60	2 00	50 00	90		0

These Tables are calculated for each of the Rumbs.

The first seven have three Columns, and of them the first containeth the degrees of Latitude from the Equinoctial to the Pole: the second doth give the difference of Longitude; and the third the distance, both of them belonging to that Rumb and Latitude.

As in the Table of the third Rumb; at the Latitude of 50 *gr.* I find under the title of Longitude 38 *gr.* 69 *parts*, and under the title of Distance 60 *gr.* 13 *parts*. This shews that if the course held constantly on the third Rumb from the Equinoctial to the Latitude of 50 *gr.* the difference of Longitude would be 38 *gr.* 69 *parts* of 100, and the distance upon the Rumb 60 *gr.* 13 *parts*. For here I reckon the distance by degrees, rather than by Leagues or Miles, and subdivided each degree into 100 *parts*, rather than into 60 minutes, for the more ease in Calculation, and withal to make the Calculation to agree the better, both with his, and my *Cross-staff* and other Instruments.

The use of these Tables, for the finding of the difference of Longitude, is this. Turn to the Table of the Rumb, and there see what Longitude belongeth to either Latitude, then take the one Longitude out of the other, the Remainder will be the difference of Longitude required.

As in the former example, where the places given were A, in the Latitude of 50 *gr.* C in the Latitude of 55 *gr.* and the Rumb the third from the Meridian: I look into the Table of the third Rumb and and there find,

Latitude 50 *gr.*	Longitude 38 *gr.* 69 *parts.*
Latitude 55 *gr.*	Longitude 44 *gr.* 19.
Therefore the difference of	Longitude 5 *gr.* 50.

There is another Use of these Tables, for the describing of the Rumbs both on the Globe, and all sorts of Charts. For having drawn the Circles of Longitude and Latitude, and finding by the Tables, the the difference of Longitude belonging to each Rumb and Latitude: If we make a prick in the Chart, at every degree of Latitude, according to that difference of Longitude, and draw Lines through those Pricks, so as they make no Angles, the Lines so drawn shall be the Rumbs required.

The Use of the Eighth Rumb is something different from the rest.

For

For there being here no change of Latitude, I have ſet to each Latitude, the difference of Longitude, belonging to one degree of diſtance, and the diſtance belonging to one degree of Longitude.

As if two places ſhall be 20 Leagues, or one degree diſtant one from the other, in the Latitude of 50 *gr.* the difference of Longitude between them will be 1 *gr.* 55 *parts.* But if they differ one degree in Longitude, the diſtance between them will be only 64 parts, which fall ſhort of 13 Leagues, or at the moſt 64 *gr.* 28 parts, ſuch as 10000 do make a degree.

6. By the difference of Longitude, Rumb, and one Latitude, to find the other Latitude.

As if the places given were A, in the Latitude of 50 *gr.* C in a greater Latitude, but unknown, the difference of Longitude 5 *gr.* ½, and the Rumb the third from the Meridian.

In the Chart let A B, D C, Meridians, be drawn through A and C, according to the difference of Longitude, one 5 *gr.* ½ from the other; and a Parallel of Latitude through A, croſſing the Meridian C D in D: then in A, with A B, make an Angle of the Rumb B A E: ſo the degrees in the Meridian between D and C, ſhall be found to be 5 *gr.* the proper difference of Latitude which was required. Wherefore the proportion holds for the Sector.

As A D the Radius,
to D C the Tangent of the Rumb from the Equator;
So A D the difference of Longitude,
to D C the proper difference of Latitude.

According to this, I take 56 *gr.* 15 *m.* for the Angle of the Rumb from the Equator, out of the greater *Tangents*, and make it a Parallel Radius. Then I reckon 5 *gr.* ½ in the Line of *Lines* from the Center, for the difference of Longitude. So the Parallel taken from the terms of this difference, and meaſured in the Line of Meridians, ſhall reach from 50 *gr.* the Latitude given, to 55 *gr.* which is the Latitude required.

Or if the Rumb fall nearer to the Meridian,

The seventh Rumb from the Meridian. | East and by North, West and by North, | East and by South, West and by South.

La	Long	Dist.	La	Long.	Dist.	La	Long.	Dist.
Gr	Gr. Par.	Gr. Par.	Gr	Gr. Par.	Gr. Par.	Gr	Gr. Par.	Gr. Par.
0	0	0	30	158 23	153 77	60	379 35	307 55
1	05 02	5 12	31	64 00	158 90	61	389 56	312 07
2	10 05	10 25	32	169 96	164 02	62	400 10	317 80
3	15 08	15 38	33	175 92	169 15	63	410 98	322 93
4	20 12	20 50	34	181 95	174 28	64	422 26	328 05
5	25 16	25 63	35	188 04	179 40	65	433 94	333 18
6	30 21	30 75	36	194 22	184 53	66	446 03	338 30
7	35 27	35 88	37	200 48	189 65	67	458 66	343 43
8	40 34	41 00	38	206 82	194 78	68	471 80	348 55
9	45 42	46 13	39	213 24	199 90	69	485 52	353 68
10	50 52	51 26	40	219 76	205 03	70	499 89	358 81
11	55 63	56 38	41	226 37	210 16	71	514 94	363 93
12	60 77	61 51	42	233 08	215 28	72	530 79	369 06
13	65 92	66 63	43	239 90	220 41	73	547 52	374 18
14	71 09	71 76	44	246 84	225 53	74	565 22	379 31
15	76 28	76 88	45	253 89	230 66	75	584 03	384 43
16	81 50	82 01	46	261 05	235 79	76	604 13	389 56
17	86 75	87 14	47	268 36	240 91	77	625 67	394 69
18	92 02	92 26	48	275 80	246 04	78	648 91	399 81
19	97 31	97 39	49	283 40	251 16	79	674 15	404 94
20	102 64	102 51	50	291 13	256 29	80	701 75	410 06
21	108 01	107 64	51	299 03	261 41	81	732 25	415 19
22	113 42	112 77	52	307 11	266 54	82	766 30	420 32
23	118 87	117 89	53	315 37	271 66	83	804 86	425 44
24	124 35	123 02	54	323 82	276 79	84	849 38	430 57
25	129 87	128 14	55	332 48	281 92	85	901 98	435 69
26	135 44	133 27	56	341 36	287 04	86	966 31	440 82
27	141 05	138 40	57	350 47	292 17	87	1049 26	445 94
28	146 71	143 52	58	359 81	297 30	88	1166 11	451 07
29	152 44	148 65	59	369 45	302 43	89	1366 23	456 20
30	158 23	158 23	60	379 35	307 55	90		

The eighth Rumb of East and West, with the Longitude answering to one deg. of distance, and the distance belonging to one degree of Longitude.

La Gr	Long. Gr. Par.	Dist. Parts.	La Gr	Long. Gr. Par.	Dist. Parts.	La Gr	Long. Gr. Par.	Dist. Parts.
0	0	10000	30	1 25	86 60	00	2 00	50 00
1	1 00	99 98	31	1 17	85 71	61	2 06	48 48
2	1 00	99 94	32	1 18	84 80	62	2 13	46 94
3	1 00	99 86	33	1 19	83 86	63	2 20	45 40
4	1 00	99 75	34	1 21	82 90	64	2 28	43 83
5	1 00	99 62	35	1 22	81 91	65	2 37	42 26
6	1 01	99 45	36	1 24	80 90	66	2 46	40 67
7	1 01	99 25	37	1 25	79 86	67	2 56	39 07
8	1 01	99 02	38	1 27	78 80	68	2 67	37 46
9	1 01	98 76	39	1 29	77 71	69	2 79	35 83
10	1 02	98 48	40	1 31	76 60	70	2 92	34 20
11	1 02	98 16	41	1 33	75 47	71	3 07	32 55
12	1 02	97 81	42	1 35	74 31	72	3 24	30 90
13	1 03	97 43	43	1 37	73 13	73	3 42	29 23
14	1 03	97 03	44	1 39	71 93	74	3 63	27 56
15	1 03	96 59	45	1 41	70 71	75	3 86	25 88
16	1 04	96 12	46	1 44	69 46	76	4 13	24 19
17	1 04	95 63	47	1 47	68 20	77	4 44	22 49
18	1 05	95 10	48	1 49	66 91	78	4 81	20 79
19	1 06	94 55	49	1 52	65 60	79	5 24	19 08
20	1 06	93 97	50	1 55	64 28	80	5 76	17 36
21	1 07	93 35	51	1 59	62 63	81	6 39	15 64
22	1 08	92 72	52	1 62	61 56	82	7 18	13 91
23	1 09	92 05	53	1 66	60 18	83	8 20	12 18
24	1 09	91 35	54	1 70	58 77	84	9 57	10 45
25	1 10	90 63	55	1 74	57 35	85	11 47	8 71
26	1 11	89 88	56	1 79	55 92	86	14 33	6 97
27	1 12	89 10	57	1 84	54 46	87	19 11	5 23
28	1 13	88 29	58	1 89	52 99	88	28 65	3 49
29	1 14	87 46	59	1 94	51 50	89	57 30	1 74
30	1 15	86 60	60	2 00	50 00	90		0

These Tables are calculated for each of the Rumbs.

The first seven have three Columns, and of them the first containeth the degrees of Latitude from the Equinoctial to the Pole: the second doth give the difference of Longitude; and the third the distance, both of them belonging to that Rumb and Latitude.

As in the Table of the third Rumb; at the Latitude of 50 *gr.* I find under the title of Longitude 38 *gr.* 69 *parts*, and under the title of Distance 60 *gr.* 13 *parts.* This shews that if the course held constantly on the third Rumb from the Equinoctial to the Latitude of 50 *gr.* the difference of Longitude would be 38 *gr.* 69 *parts* of 100, and the distance upon the Rumb 60 *gr.* 13 *parts.* For here I reckon the distance by degrees, rather than by Leagues or Miles, and subdivided each degree into 100 *parts*, rather than into 60 minutes, for the more ease in Calculation, and withal to make the Calculation to agree the better, both with his, and my *Cross-staff* and other Instruments.

The use of these Tables, for the finding of the difference of Longitude, is this. Turn to the Table of the Rumb, and there see what Longitude belongeth to either Latitude, then take the one Longitude out of the other, the Remainder will be the difference of Longitude required.

As in the former Example, where the places given were A, in the Latitude of 50 *gr.* C in the Latitude of 55 *gr.* and the Rumb the third from the Meridian: I look into the Table of the third Rumb and and there find,

Latitude 50 *gr.*	Longitude 38 *gr.* 69 *parts.*
Latitude 55 *gr.*	Longitude 44 *gr.* 19.
Therefore the difference of	Longitude 5 *gr.* 50.

There is another Use of these Tables, for the describing of the Rumbs both on the Globe, and all sorts of Charts. For having drawn the Circles of Longitude and Latitude, and finding by the Tables, the the difference of Longitude belonging to each Rumb and Latitude: If we make a prick in the Chart, at every degree of Latitude, according to that difference of Longitude, and draw Lines through those Pricks, so as they make no Angles, the Lines so drawn shall be the Rumbs required.

The Use of the Eighth Rumb is something different from the rest.

For

For there being here no change of Latitude, I have ſet to each Latitude, the difference of Longitude, belonging to one degree of diſtance, and the diſtance belonging to one degree of Longitude.

As if two places ſhall be 20 Leagues, or one degree diſtant one from the other, in the Latitude of 50 *gr.* the difference of Longitude between them will be 1 *gr.* 55 *parts.* But if they differ one degree in Longitude, the diſtance between them will be only 64 parts, which fall ſhort of 13 Leagues, or at the moſt 64 *gr.* 28 parts, ſuch as 10000 do make a degree.

6. *By the difference of Longitude, Rumb, and one Latitude, to find the other Latitude.*

As if the places given were A, in the Latitude of 50 *gr.* C in a greater Latitude, but unknown, the difference of Longitude 5 *gr.* ½, and the Rumb the third from the Meridian.

In the Chart let A B, D C, Meridians, be drawn through A and C, according to the difference of Longitude, one 5 *gr.* ½ from the other; and a Parallel of Latitude through A, croſſing the Meridian C D in D: then in A, with A B, make an Angle of the Rumb B A E: ſo the degrees in the Meridian between D and C, ſhall be found to be 5 *gr.* the proper difference of Latitude which was required. Wherefore the proportion holds for the *Seſter*,

As A D the Radius,
to D C the Tangent of the Rumb from the Equator;
So A D the difference of Longitude,
to D C the proper difference of Latitude.

According to this, I take 56 *gr.* 15 *m.* for the Angle of the Rumb from the Equator, out of the greater *Tangent*, and make it a Parallel Radius. Then I reckon 5 *gr.* ½ in the Line of *Lines* from the Center, for the difference of Longitude. So the Parallel taken from the terms of this difference, and meaſured in the Line of Meridians, ſhall reach from 50 *gr.* the Latitude given, to 55 *gr.* which is the Latitude required.

Or if the Rumb fall nearer to the Meridian,

As B C the Tangent of the Rumb from the Meridian,
is to A B the Radius:
So B C the difference of Longitude,
to A D the proper difference of Latitude.

According to this we may best work by Parallel entrance; first take 35 *gr.* 45 *m.* for the Angle of the Rumb from the Meridian, out of the greater Tangent, and make it a Parallel Radius; then take 5 *gr.* ½ for the difference of Longitude out of the Line of *Lines*, and carry it Parallel to the former, till the feet of the Compasses stay in like Points: so the Line between the Center and the place of this stay, being taken and measured in the Line of Meridians from 50 *gr.* forward, shall shew the Latitude required to be 55 *gr.* as in the former way.

The like may be found by the Tables of Rumbs. For in the Table of the third Rumb, at the Latitude of 50 *gr.* I find the Longitude of 38 *gr.* 69 *p.* To this if I add 5 *gr.* 50 *p.* for the difference of Longitude given, the compound Longitude will be 44 *gr.* 19 *p.* and this answers to the Latitude of 55 *gr.*

But if this difference of Latitude were to be found by the common Sea-chart, it should seem to be 8 *gr.* 13 *m.* and so the second Latitude should be 58 *gr.* 13 *m.* which is above 3 *gr.* more than the truth.

7. *By one Latitude, Rumb, and distance, to find the difference of Longitude.*

As if the places given were A in the Latitude of 50 *gr.* C in a greater Latitude but unknown, the distance upon the Rumb be 6 *gr.* between them, and the Rumb the third from the Meridian.

In the Chart, let a Meridian A B, and a Parallel A D, be drawn through A, and in A, with A B, make an Angel B A C, for the Rumb from the Meridian; then open the Compasses according to the Latitude of the places to E F, the quantity of 6 *gr.* in the Meridian, transferring them into the Rumb from A to C, and through C draw another Meridian D C, crossing the Parallel drawn through A in D, so the degrees intercepted in the Parallel from A to D, shall shew the difference of Longitude required to be about 5 *gr.* ½. Wherefore the proportion holds for the Sector.

As

As A C the Radius,
is to A D, equal to B C, the Sine of the Rumb from the Meridian:
So A C the proper diſtance upon the Rumb,
to A D the difference of Longitude.

According to this I take the Sine of 33 *gr.* 45 *m.* for the Angle of the Rumb from the Meridian, and make it a Parallel Radius; then keeping the *Sector* at this Angle, I take 6 *gr.* for the diſtance, out of the Meridian Line, according to the eſtimated Latitudes of both places, and lay it on both ſides of the *Sector* from the Center: ſo the Parallel taken from the terms of this diſtance, and meaſured in the Lines of *Lines*, ſhall ſhew the difference of Longitude to be about 5 *gr.* ½.

In this and ſome of the *Prop.* following, where there is but one Latitude known, there may be ſometimes an error of a minute or two, in the eſtimation of the proper diſtance, yet it may be rectified at a ſecond operation.

This Propoſition may alſo be wrought by the Tables of Rumbs. For according to the Example, in the Table of the third Rumb, at the Latitude of 50 *gr.* I find the Longitude of 38 *gr.* 69 *p.* and the diſtance of 60 *gr.* 13 *p.* to this I add 6 *gr.* for the diſtance given; ſo the compound diſtance will be 66 *gr.* 13 *p.* and this anſwers to the Longitude of 44 *gr.* 19 *p.* then if I take the one Longitude out of the other, the difference will be 5 *gr.* 50 *p.* as before.

But if this difference were to be found by the common Sea-chart, it ſhould ſeem to be only 3 *gr.* 20 *m.* which is more than 2 *gr.* leſs than the truth.

8. *By one Latitude, Rumb, and difference of Longitudes, to find the diſtance.*

As if the places given were A, in the Latitude of 50 *gr.* C in a greater Latitude but unknown, the difference of Longitude between them being 5 *gr.* ½, and the Rumb the third from the Meridian.

In the Chart let A B, D C, Meridians be drawn through A and C, according to the difference of Longitude, and a Parallel of Latitude through A, croſſing the Meridian D C in D; then in A, with A B, make an Angle of the Rumb B A C; ſo the diſtance on the Rumb from

from A to C taken and measured in the Meridian, according to the estimated Latitude of the places, shall be found to be 6 *gr.* Wherefore the proportion holds for the *Sector*.

As A D, equal to B C, the Sine of the Rumb from the Meridian,
is to A C the Radius:
So A D the difference of Longitudes,
to A C the proper distance upon the Rumb:

According to this, I take the lateral Radius, and make it a Parallel Sine of 33 *gr.* 45 *m.* which is here the Angle of the Rumb from the Meridian; then I reckon 5 *gr.* ½ in the Lines of *Lines* from the Center, for the difference of Longitude: so the Parallel taken from the terms of this difference, and measured in the Line of Meridians, according to the Latitudes of the places, shall there shew the distance required to be about 6 *gr.* which are 120 Leagues.

Or if the Rumb fall nearer to the Meridian, that the lateral Radius cannot be fitted over in his Sine, this *Prop.* must be wrought by Parallel entrance, and so also it gives the same distance as before.

Or we may find this distance by the Table of Rumbs. For in the Table of the third Rumb, at the Latitude of 50 *gr.* I find the Longitude of 38 *gr.* 69 *p.* and the distance of 60 *gr.* 13 *p.* To this Longitude here found, I add 5 *gr.* 50 *p.* for the difference of Longitude given: so the compound Longitude will be 44 *gr.* 19 *p.* and this answers to the distance of 66 *gr.* 15 *p.* Then if I take the one distance out of the other, the remainder will be 6 *gr.* 2 *p.* for the distance required.

But if this distance were to be measured on the common Sea-chart, it should seem to be almost 10 *gr.* or at the least 197 Leagues, above 77 Leagues more than the truth.

9. *By one Latitude, distance, and difference of Longitude, to find the Rumb.*

As if the places given were A, in the Latitude of 50 *gr.* C in a greater Latitude, but unknown, the difference of Longitude between them being 5 *gr.* ½, and the distance 6 *gr.* upon the Rumb.

In the Chart let A B, D C, Meridians, be drawn through A and C, and

and a Parallel of Latitude through A; then open the Compaſſes according to the Latitudes of the places, to E F the quantity of 6 *gr.* in the Meridian, and ſetting the one foot in A, the other foot ſhall croſs the other Meridian in C: and if we draw the right Line A C, the Angle B A C ſhall ſhew the inclination of the Rumb to the Meridian, to be about 33 *gr.* 45 *m.* Wherefore the proportion holds for the *Sector*,

As A C the proper diſtance upon the Rumb,
is to A D the difference of Longitude:
So A C Radius,
to A D, equal to B C, the Sine of the Rumb from the Meridian.

According to this, I take the proper diſtance 6 *gr.* out of the Line of Meridians, and lay it on both ſides of the *Sector* from the Center; then I take the difference of Longitude 5 *gr.* ½ out of the Line of *Lines*, and to it open the *Sector* in the terms of the former diſtance: ſo the Parallel Radius taken from between 90 and 90, and meaſured in the Sines, doth give about 33 *gr.* 45 *m*, for the Rumb required.

But if this Rumb were to be found by the common Sea-chart, it ſhould ſeem to be above 66 *gr.* and ſo almoſt the ſixth Rumb from the Meridian.

10. *By the Longitude and Latitude of two places, to find their diſtance upon the Rumb.*

Let the *Sector* be opened in the Lines of *Lines* unto a right Angle (as was ſhewed before *Cap.* 2. *Prop.* 7.) then take out the proper difference of Latitude, and lay it on the one Line, and the difference of Longitude, and lay it on the other line, ſo as they may both meet in the Center, marking how far they extend. For the Line taken from the terms of their extenſion, and meaſured in the Meridian, according to their Latitudes, ſhall ſhew the diſtance required.

So if the places given were A and C, A in the Latitude of 50 *gr.* C in the Latitude of 55 *gr.* the proper difference of Latitude ſhall be the Line A B, and let B C the difference of Longitude be 5 *gr.* ½. we ſhall find that A C the diſtance upon the Rumb is about 6 *gr.* which make 120 Leagues.

For in the Chart, let an occult Meridian be drawn through A, and a Parallel of Latitude through C, croſſing the former Meridian in B, and a right Line for the Rumb, from A to C, ſo have we a Rectangle Triangle A B C, whoſe Baſe A C, taken and meaſured in the Meridian from E below 50 *gr.* to F, as much above 55 *gr.* doth contain the quantity of 6 *gr.*

In the ſame manner the *Sector* being opened to a right Angle, in the Lines of *Lines*; if we take the difference of Latitude out of the Line of Meridians, in his proper place from 50 *gr.* to 55 *gr.* and place it on one of the ſides from the Center, to reſemble A B, then reckon the difference of Longitude on the other Perpendicular Line from the Center to 5 *gr.* ½, inſtead of B C, we ſhall have the like Rectangle Triangle on the *Sector*, to that which we had before on the Chart; and if we take out the Baſe of it, and meaſure it in the Line of Meridians from below 5 *gr.* to as much above 55 *gr.* we ſhall find as before, that it containeth about 6 *gr.* or 120 Leagues.

But if this diſtance were to be meaſured on the common Sea-chart, it ſhould ſeem to be almoſt 7 *gr.* ¼, or 245 Leagues; which is 25 *leagues* more than the truth.

11. *By the Latitude of two places, and the diſtance upon the Rumb, to find the difference of Longitude.*

Let the *Sector* be opened in the Lines of *Lines* to a right Angle, then take out the proper difference of Latitudes, and lay it on one of the Lines from the Center, then take the proper diſtance with a pair of Compaſſes, and ſetting one foot in the terms of the difference, turn the other foot to the other Line of the *Sector*, and it ſhall there ſhew the difference of Longitude required.

So if the place given were A, in the Latitude of 50 *gr.* C in the Latitude of 55 *gr.* with 6 *gr.* of diſtance one from another, we ſhall find their difference of Longitude to be about 5 *gr.* ½.

For in the Chart let a Meridian A B be drawn for the one, and A C, A D, Parallels of Latitude for them both: Then open the Compaſſes according to the Latitude of the places, to E F the quantity of 6 *gr.* in the Meridians, and ſetting one foot in A, having Latitude of 50 *gr.* turn the other to the Parallel of 55 *gr.* and it ſhall there cut off the required difference of Longitude B C 5 *gr.* ½.

In

In the ſame manner, the *Sector* being opened to a right Angle, in the Lines of *Lines:* if we take the difference of Longitude out of the Line of Meridians in his proper place from 50 *gr.* unto 55 *gr.* and place it on one of the Lines from the Center; then take 6 *gr.* the diſtance upon the Rumb out of the ſame Line of Meridians, according to the Latitudes of the places, and ſet the one foot in the term of the former difference, turning the other foot to the other Perpendicular Line, we ſhall find that it will croſs it about 5 *gr.* $\frac{1}{2}$ from the Center, which is the difference of *Longitude* required.

But if this difference of *Longitude* were to be found by the common Sea-chart, it would ſeem to be only 3 *gr.* 20 *m.* which is more than 2 *gr.* 10 *m.* leſs than the truth.

12. *By one Latitude, diſtance and difference of Longitudes, to find the difference of Latitudes.*

Let the *Sector* be opened in the *Line* of *Lines* to a right Angle, and let the difference of *Longitude* be reckoned in one of thoſe *Lines* from the Center; then take the proper diſtance with a pair of Compaſſes, and ſetting the one foot in the term of the former difference, turn the other foot to the other *Line* of the *Sector*, and it ſhall thence cut off a *Line*, equal to the proper difference of *Latitude* required.

So if the places given were A and C, A in the Latitude of 50 *gr.* C in a greater *Latitude* but unknown, the difference of *Longitude* between them 5 *gr.* $\frac{1}{2}$, and the diſtance upon the Rumb 6 *gr.* or 120 Leagues, we ſhall find the difference of *Latitude* to be 5 *gr.*

For in the Chart, let occult Meridians be drawn through A and C, and a Parallel of *Latitude* through A, then open the Compaſſes according to the eſtimated *Latitudes* of the places to E F the quantity of 6 *gr.* in the Meridian, and ſetting the one foot in A, turn the other to the Meridian drawn through C, and it ſhall there cut off the *Line* D C, which is the difference of *Latitude* required.

In the ſame manner, the *Sector* being opened to a right Angle in the *Line* of *Lines*, if in the one *Line* we reckon the difference of *Longitude* from the Center to 5 *gr.* $\frac{1}{2}$, then taking 6 *gr.* for the diſtance out of the *Line* of Meridians, according to the *Latitude* of the places, we ſet the one foot in the term of the given difference, and turn the other foot to the other Perpendicular *Line*, we ſhall find that

it cuts a *Line* from it, which taken and meaſured in the *Line* of Meridians, from 50 *gr.* on forward, doth ſhew the difference of *Latitude* to be as before 5 *gr.*

But if this difference of *Latitude* were to be found by the common Sea-chart, it would ſeem to be only 2 *gr.* 25 *m.* which is 2 *gr.* 35 *m.* leſs than the truth. Such is the difference between both theſe Charts.

THE

THE THIRD BOOK OF THE SECTOR,

Containing the Uſe of the particular Lines.

THe *Lines* of *Lines*, of Superficies, of Solids, of Sines, with the lateral *Lines* of Tangents and Meridians, whereof I have hereunto ſpoken, are thoſe which I principally intended; that little room on the *Sector* which remaineth, may be filled up with ſuch particular *Lines*, as each one ſhall think convenient for his purpoſe. I have made choice of ſuch as I thought might be beſt prickt on without hindring the ſight of the former, *viz.* Lines of *Quadrature*, of *Segments*, of *Inſcribed bodies*, of *Equated bodies*, and of *Metals*.

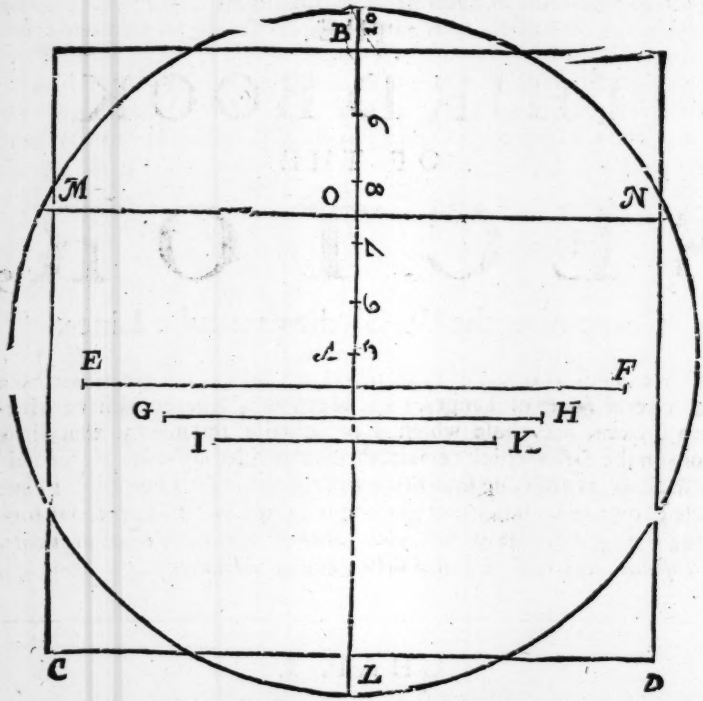
CHAP. I.

Of the Lines of Quadrature.

THe Lines of Quadrature may be known by the letter Q, and by their place between the Lines of Sines. Q ſignifieth the ſide of a Square; 5 the ſide of a Pentagon with five equal ſides, 6 of a Hexagon with ſix equal ſides, and ſo 7, 8, 9, and 10. S ſtands for the Semidiameter of a Circle, and 90 for a Line equal to 90 *gr.* in the Circumference. The uſe of them may be:

1. *To make a ſquare equal to a Circle given:*
2. *To make a Circle equal to a Square given.*

If



If the Circle be firſt given, take his Semidiameter; and to it open the *Sector* in the Points at S: ſo the Parallel taken from between the Points at Q, ſhall be the ſide of the Square required.

If the Square be given, take his ſide, and to it open the *Sector*, in the Points at Q: So the Parallel taken from between the Points at S, ſhall be the Semidiameter of the Circle required.

Let the Semidiameter of the Circle given be A B, the ſide of the Square equal unto it ſhall be found to be C D.

3. *To reduce a Circle given, or a Square into an equal Pentagon, or other like ſided and like angled Figure.*

Take the ſide of the Figure given, and fit it over in his due Points: ſo

ſo the Parallels taken from between the Points of the other Figures, ſhall be the ſides of thoſe Figures: which being made up with Equal Angles, ſhall be all equal one to the other.

Let the Semidiameter of the Circle given be A B, the ſide of an *Hexagon* equall to this Circle, ſhall by theſe means be found to be G H; and the ſides of an Octagon to be I K. Other Planes not here ſet down, may firſt be reduced into a Square, by the ſixth *Prop. Superf.* and then into a Circle or other of theſe equal Figures, as before.

4. *To find a right Line, equal to the Circumference of a Circle, or other part thereof.*

Take the Semidiameter of the Circle given, and to it open the *Sector* in the Points at S; ſo the Parallel taken from between the Points at 90 in this Line, ſhall be the fourth part of the Circumference: which being known, the other parts may be found out by the ſecond and third *Prop.* of Lines.

Thus if the Semidiameter of the Circle given be A B, the right Line E F ſhall be found to be the fourth part of the Circumference. Therefore the double of E F ſhall be equal to the Circumference of 180 *gr.* and the half of E F be the Circumference of 45 *gr.* and ſo in the reſt.

CHAP. II.

Of the Lines of Segments.

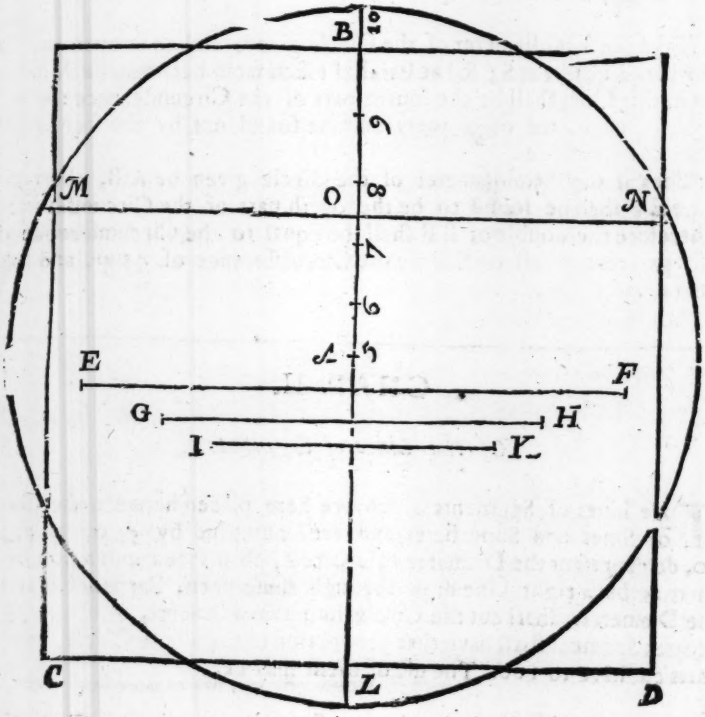
THe Lines of Segments which are here placed between the Lines of Sines and Superficies and are numbred by 5, 6, 7, 8, 9, 10, do repreſent the Diameter of a Circle, ſo divided into a hundred parts, as that a right Line draw through theſe parts, Perpendicular to the Diameter, ſhall cut the Circle into two Segments, of which the greater Segment ſhall have that proportion to the whole Circle, as the parts cut have to 100. The uſe of them may be,

1. *To divide a Circle given into two Segments according to a Proportion given.* 2. *To*

2. To find a Proportion between a Circle and his Segments given.

Let the *Sector* be opened in the Points of 100 to the Diameter of the Circle given: ſo a Parallel taken from the Points proportionable to the great Segment required, ſhall give the depth of that greater Segment.

Or if the Segments be given, let the *Sector* be opened as before; then take the depth of the greater Segment, and carry it Parallel to the Diameter: ſo the number of Points wherein they ſtay, ſhall ſhew the proportion to 100.



As if the Diameter of the Circle given were B L, the depth of the greater Segmeat L O being 75, doth ſhew the proportion of the Segment O M L N to the Circle, to be as 75, to 100; *viz.* three parts of four.

Hence I might ſhew, if there were any uſe of it,

2. *To find the ſide of a Square, equal to any known Segment of a Circle.*

The ſide of a Square equal to the whole Circle, may be found by the former *Chap.* and then having the proportion of the Segment to the Circle, we may diminiſh the Square in ſuch proportion by that which hath been ſhewed *Lib.*1. *Cap.*3. *Prop.*3.

CHAP. III.

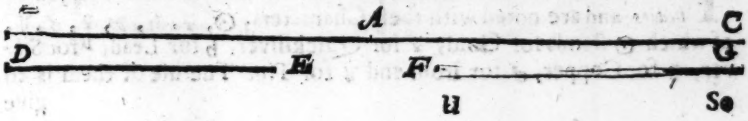
Of the Lines of Inſcribed Bodies.

THe Lines of *Inſcribed Bodies* are here placed between the Lines of *Lines*, and may be known by the letters D,S,I,C,O,T, of which D ſignifieth the ſide of a *Dodecahedron*, I of an *Icoſahedron*, C of a *Cube*, O of an *Octahedron*, and T of a *Tetrahedron*, all inſcribed into the ſame Sphere, whoſe Semidiameter is here ſignified by the letter S.

The uſe of theſe Lines may be,

1. *The Semidiameter of a Sphere being given, to find the ſides of the five regular Bodies, which may be inſcribed in the ſaid Sphere.*
2. *The ſide of any of the five regular Bodies being given, to find the Semidiameter of a Sphere, that will circumſcribe the ſaid Body.*

If the Sphere be firſt given, take his Semidiameter, and to it open the *Sector* in the Points at S: if any of the other bodies be firſt given, take the ſide of it, and fit it over in his due Points: ſo the Parallel taken from between the Points of the other bodies, ſhall be the ſides of thoſe bodies, and may be inſcribed into the ſame Sphere.



So if the Semidiameter of the Sphere be A C, the ſide of the *Dodecahedron* inſcribed ſhall be D E.

CHAP. IV.

Of the Lines of Equated bodies.

THe Lines of *Equated bodies*, are here placed between the Lines of *Lines* and *Solids*, noted with theſe letters D, I, C, S, O, T, of which D ſtands for the ſide of a *Dodecahedron*, I for the ſide of an *Icoſahedron*, C for the ſide of a *Cube*, S for the Diameter of a *Sphere*, O for the ſide of an *Octahedron*, and T for the ſide of a *Tetrahedron*, all equal one to the other. The uſe of theſe Lines may be,

1. *The Diameter of a Sphere being given, to find the ſides of the five regular bodies, equal to that Sphere.*
2. *The ſide of any of the five regular bodies being given, to find the Diameter of a Sphere, and the ſides of the other bodies, equal to the firſt body given.*

If the Sphere be firſt given, take its Diameter, and to it open the *Sector* in the Points at S: if any of the other bodies be firſt given, take the ſide of it, and fit it over in his due Points, ſo the Parallels taken from between the Points of the other bodies, ſhall be the ſides of thoſe bodies equal to the firſt body given.

Thus in the laſt Diagram, if the Diameter of a Sphere given be B C, the ſide of the *Dodecahedron* equal to this Sphere, would be found to be F G.

CHAP. V.

Of the Lines of Metals.

THe Lines of *Metals* are here joyned with thoſe before of *Equated bodies*, and are noted with theſe Characters, ☉, ☿, ♄, ☽, ♀, ♂, ♃, of which ☉ ſtands for Gold, ☿ for Quickſilver, ♄ for Lead, ☽ for Silver, ♀ for Copper, ♂ for Iron, and ♃ for Tin. The uſe of them is to

give

give a proportion beween theſe ſeveral Metals, in their magnitude and weight, according to the experiments of *Marinus Ghetaldus*, in his book called *Promotus Archimedes*.

1. *In like bodies of ſeveral Metals, and equal weight, having the magnitude of the one, to find the magnitude of the reſt.*

Take the magnitude given out of the Lines of Solids, and to it open the *Sector* in the Points belonging to the Metal given: ſo the Parallels taken from between the Points of the other Metals, and meaſured in the Lines of Solids, ſhall give the magnitude of their bodies.

Thus, having Cubes or Spheres of equal weight, but ſeveral Metals, we ſhall find, that if thoſe of Tin contain 10000 D, the others of Iron will contain 9250, thoſe of Copper 8222, thoſe of Silver 7161, theſe of Lead 6435, thoſe full of Quickſilver 5493, and thoſe of Gold 3895.

2. *In like bodies of ſeveral Metals and equal magnitude, having the weight of one, to find the weights of the reſt.*

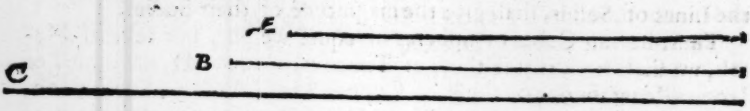
This Propoſition is the converſe of the former, the proportion not direct, but reciprocal, wherefore having two like bodies, take the given weight of the one out of the Lines of *Solids*, and to it open the *Sector* in the Points belonging to the Metal of the other body: ſo the Parallel taken from the Points belonging to the body given, and meaſured in the Lines of *Solids*, ſhall give the weight of the body required.

As if a Cube of Gold weighed 38 *l.* and it were required to know the weight of a Cube of Lead having equal magnitude. Firſt I take 38 *l.* for the weight of the golden Cube out of the Lines of *Solids*, and put it over in the Points of ♄ belonging to Lead: ſo the Parallel taken from between the Points of ☉ ſtanding for Gold, and meaſured in the Lines of Solids, doth give the weight of the leaden Cube required to be 13 *l.*

Thus if a Sphere of Gold ſhall weigh 10000, we ſhall find that a Sphere of the ſame Diameter full of Quickſilver ſhall weigh 7143, a Sphere of Lead 6053, a Sphere of Silver 5438, a Sphere of Copper 4737, a Sphere of Iron 4210, and a Sphere of Tin 3895.

3. *A body being given of one Metal, to make another like unto it of another Metal, and equal weight.*

Take out one of the ſides of the body given, and put it over in the Points belonging to his Metal: ſo the Parallel taken from between the Points belonging to the other metal, ſhall give the like ſide, for the body required. If it be an irregular body, let the other like ſides be found out in the ſame manner.



Let the body given be a Sphere of Lead containing in Magnitude 16 *d*, whoſe Diameter is A, to which I am to make a Sphere of Iron, of equal weight: If I take out the Diameter A, and put it over in the Points of ♄ belonging to Lead, the Parallel taken from between the Points of ♂, ſtanding for Iron, ſhall be B, the Diameter of the Iron Sphere required. And this compared with the other Diameter, in the Lines of *Solids*, will be found to be 23 *d.* in magnitude.

4. *A body being given of one Metal, to make another like unto it of another Metal, according to a weight given.*

Firſt, find the ſides of a like body of equal weight, then may we either augment or diminiſh them according to the proportion given, by that which we ſhewed before in the ſecond and third *Prop.* of *Solids.*

As if the body given were a Sphere of Lead, whoſe Diameter is A, and it were required to find the Diameter of a Sphere of Iron, which ſhall weigh three times as much as the Sphere of Lead: I take A, and put it over in the Points of ♄, his Parallel taken from between the Points of ♂, ſhall give me B for the Diameter of an equal Sphere of Iron: if this be augmented in ſuch proportion as 1 unto 3, it giveth C, for the Diameter required.

CHAP.

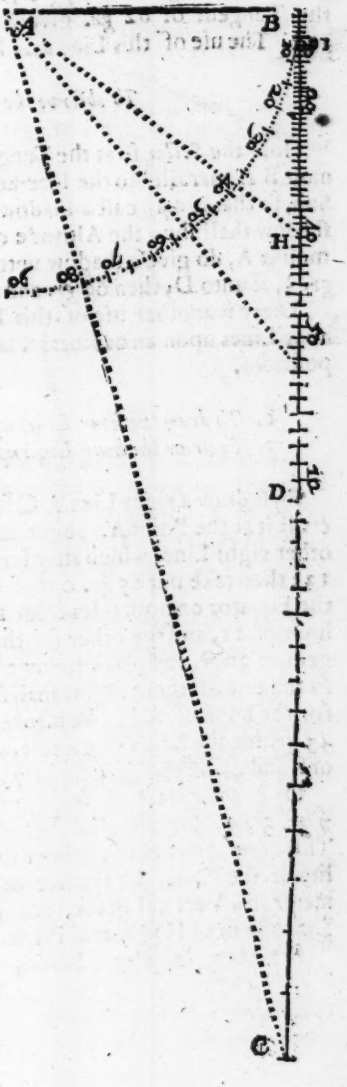
CHAP. VI.

Of the Lines on the edges of the Sector.

HAving ſhewed ſome uſe of the Lines on the flat ſides of the *Sector*, there remain only thoſe on the edges. And here one half of the outward edge is divided into inches, and numbred according to their diſtance from the ends of the *Sector*. As in the Sector of fourteen inches long, where we find 1 and 13, it ſheweth that diviſion to be 1 inch from the nearer end, and 13 inches from the farther end of the *Sector*.

The other half containeth a Line of leſſer Tangents, to which the Gnomon is Radius; They are here continued to 75 *gr.* And if there be need to produce them farther, Take 45 *d*, out of the number of degrees required, and double the remainder: ſo the Tangent and Secant of this double remainder being added, ſhall make up the Tangent of the degrees required.

As if A B being the Radius, and B C the Tangent Line, it were required to find the Tangent of 75 *gr.* If we take 45 *gr.* out of 75 *gr.* the remainder is 30 *gr.* and the double 60 *gr.* whoſe Tangent is B D, and the Secant is A D: if then we add A D to B D, it maketh B C, the Tangent of 75 *gr.* which was required. In like ſort, the Secant of 61 *gr.* added to the Tangent of 61 *gr.* giveth



giveth the Tangent of 75 *gr.* 30 *m.* and the Secant of 62 *gr.* added to the Tangent of 62 *gr.* giveth the Tangent of 76 *gr.* and ſo in the reſt. The uſe of this Line may be,

To obſerve the Altitude of the Sun.

Hold the *Sector* ſo as the Tangent B C, may be Vertical, and the Gnomon B A, parallel to the Horizon; then turn the Gnomon toward the Sun, ſo that it may caſt a ſhadow upon the Tangent, and the end of the ſhadow ſhall ſhew the Altitude of the Sun. So if the end of the Gnomon at A, do give a ſhadow unto H, it ſheweth that the Altitude is 38 *gr.* ½, if unto D, then 60 *gr.* and ſo in the reſt.

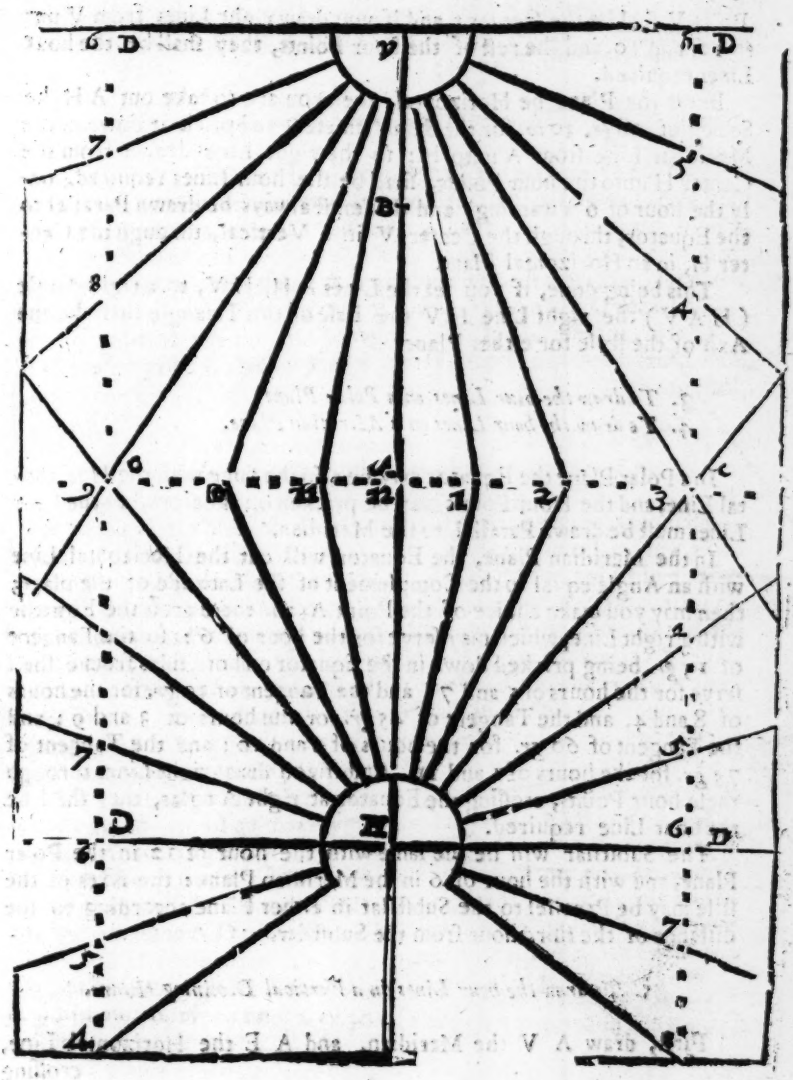
There is another uſe of this Tangent Line, for the drawing of the hour Lines upon an ordinary Plane, whereof I will ſet down theſe Propoſitions.

1. *To draw the hour Lines upon an Horizontal Plane.*
2. *To draw the hour Lines upon a direct Vertical Plane.*

Firſt draw a right Line A C for the Horizon, and the Equator, and croſs it at the Point A, about the middle of the Line, with A B another right Line, which may ſerve for the Meridian, and the hour of 12; then take out 15 *gr.* out of the Tangents, and prick them down in the Equator on both ſides from 12: ſo the one Point ſhall ſerve for the hour of 11, and the other for the hour of 1. Again, take out the Tangent of 30 *gr.* and prick it down in the Equator on both ſides from 12: ſo the one of theſe Points ſhall ſerve for the hour of 10, and the other for the hour of 2. In like manner may you prick down the Tangent of 45 *gr.* for the hours of 9 and 3, and the Tangent of 60 *gr.* for the hours of 8 and 4, and the Tangent of 75 *gr.* for the hours of 7 and 5.

Or if any pleaſe to ſet down the parts of an hour, he may allow 7 *gr.* 30 *m.* for every half hour, and 3 *gr.* 45 *m.* for every quarter. This done, you are to conſider the Latitude of the place, and the quality of the Plane. So the Secant of the Latitude ſhall be the Semidiameter in a Vertical Plane, and the Secant of the Complement of the Latitude in an Horizontal Plane.

For example, about *London*, the Latitude is 51 *gr.* 30 *m.* and let the Plane be Vertical. If you take A V, the Secant of 51 *gr.* 30 *m.* out of the *Sector*, and prick it down in the Meridian Line from A to V, the Point


6 D
V
6 D
7
5
B
8
4
C
A
C
9
10
11
12
1
2
3
7
5
D
H
6 D
6
7
5
4

Point V ſhal be the Center: and if you draw right Lines from V unto 11, and 10, and the reſt of the hour Points, they ſhall be the hour Lines required.

But if the Plane be Horizontal, then you are to take out A H the Secant of 38 *gr.* 30 *m.* for the Semidiameter, and prick it down in the Meridian Line from A unto H; ſo the right Lines drawn from the Center H unto the hour Points, ſhall be the hour Lines required; only the hour of 6 is wanting, and that muſt always be drawn Parallel to the Equator, through the Center V in a Vertical, through the Center H, in an Horizontal Plane.

This being done, if you ſet the Lines A H, H V, to a right Angle (H A V) the right Line H V the Baſe of this Triangle ſhall be the Axis of the ſtyle for either Plane.

3. *To draw the hour Lines on a Polar Plane.*
4. *To draw the hour Lines on a Meridian Plane.*

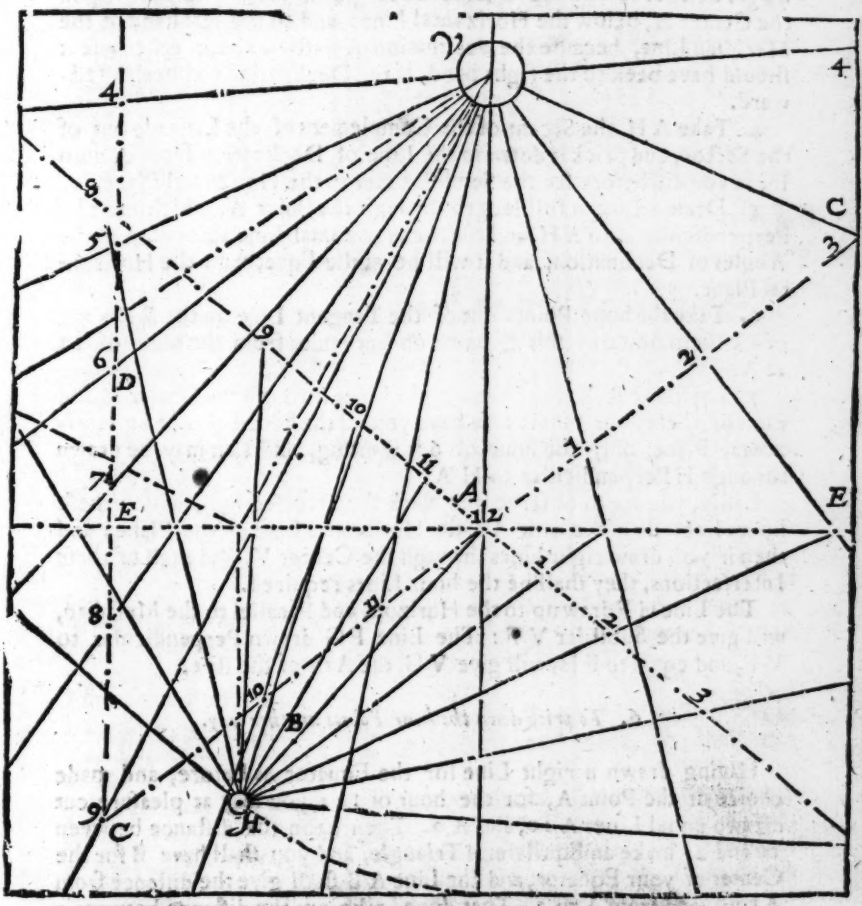
In a Polar Plane the Equator may be alſo the ſame with the Horizontal Line, and the Hour Points may be pricked on as before, but the hour Lines muſt be drawn Parallel to the Meridian.

In the Meridian Plane, the Equator will cut the Horizontal Line with an Angle equal to the Complement of the Latitude of the place; then may you make choice of the Point A, and there croſs the Equator with a right Line, which may ſerve for the hour of 6: ſo the Tangent of 15 *gr.* being pricked down in the Equator on both ſides from 6, ſhall ſerve for the hours of 5 and 7; and the Tangent of 30 *gr.* for the hours of 8 and 4, and the Tangent of 45 *gr.* for the hours of 3 and 9; and the Tangent of 60 *gr.* for the hours of 2 and 10; and the Tangent of 75 *gr.* for the hours of 1 and 11. And if you draw right Lines through theſe hour Points, croſſing the Equator at right Angles, they ſhall be the hour Line required.

The Subſtilar will be the ſame with the hour of 12 in the Polar Plane, and with the hour of 6 in the Meridian Plane: the Axes of the ſtile may be Parallel to the Subſtilar in either Plane according to the diſtance of the third hour from the Subſtilar.

5. *To draw the hour Lines in a Vertical Declining Plane.*

Firſt, draw A V the Meridian, and A E the Horizontal Line, croſſing



croſſing one the other at right Angles in the point A.

2. Then take out A V, the Secant of the Latitude of the place, which you may ſuppoſe to be 51. *gr.* 30 *m.* and prick it down in the Meridian Line from A unto V:

3. Becauſe it is a declining Plane, and you may ſuppoſe it to decline

40 *gr.* Eastward, you are to make an Angle of the Declination upon the Center A, below the Horizontal Line, and to the left hand of the Meridian Line, because the declination is Eastward, for otherwise it should have been to the right hand, if the Declination had been Westward.

4. Take A H, the Secant of the Complement of the Latitude out of the Sector, and prick it down in the Line of Declination from A unto H, as you did before for the Semidiameter in the Horizontal Plane.

5. Draw a Line at full length through the Point A, which must be Perpendicular unto A H, and cut the Horizontal Line according to the Angles of Declination, and it will be as the Equator in the Horizontal Plane.

6. Take the hour Points out of the Tangent Line in the *Sector* and prick them down in this Equator on both sides from the hour of 12 at A.

7. Lay your Ruler, and draw right Lines through the Center H, and each of these hour Points: so have you all the hour Lines of an Horizontal Plane, only the hour of 6 is wanting, and that may be drawn through H Perpendicular to H A.

Lastly, you are to observe and mark the Intersections, which these hours lines do make with A E the Horizontal Line of the Plane: and then if you draw right Lines through the Center V, and each of these Intersections, they shall be the hour Lines required.

The Line H F drawn up to the Horizon, and Parallel to the Meridian, will give the Substilar V F: The Line F G drawn Perpendicular to V F, and equal to F H, will give V G, the Axis of the stile.

6. *To prick down the hour Points another way.*

Having drawn a right Line for the Equator as before, and made choice of the Point A, for the hour of 12: you may at pleasure cut off two equal Lines A 10, and A 2. Then upon the distance between 10 and 2, make an Equilateral Triangle, and you shall have B for the Center of your Equator, and the Line A B shall give the distance from A to 9, and from A to 3. That done, take out the distance between 9 and 3, and this shall give the distance from B unto 8, and from B unto 4; again, from 4 to 11, and from 8 unto 1, and also from 8 to 7. So have you the hour Points, and if you take out the distance B 1, B 3, B 5, *&c.* You may find the Points not only for the half hours, but also for the quarters.

But

But if it so fall out, that some of these hour Points fall out of your Plane, you may help your self by the larger Tangent, both in the Vertical, and Horizontal Planes.

For if at the hour Points of 3 and 9, in the Scheme of the Horizontal and Vertical Dials, you draw occult Lines Parallel to the Meridian; the distances D C between the hour Line of 6, and the hour Points of 3 and 9, will be equal to the Semidiameter A V in a Vertical, and A H in a Horizontal Plane, and if they be divided in such sort as the Line A C is divided, you shall have the Points of 4, and 5, and 7, and 8, with their halfs and quarters.

As in the Horizontal Plane, take out the Semidiameter A H, and make it a Parallel Radius by fitting it over in the Sines of 90 and 90: Then take 15 *gr.* out of the larger Tangent and lay them on the Lines of Sines, where they will reach from the Center unto the Sines of 15 *gr.* 32 *m.* therefore take out the Parallel Sine of 15 *gr.* 32 *m.* and it shall give the distance from 6 unto 5, and from 6 unto 7, in your Horizontal Plane. That done, take out 30 *gr.* out of the larger Tangent, and lay them on the Sines, from the Center unto the Sines of 35 *gr.* 16 *m.* and the Parallel Sine of 35 *gr.* 16 *m.* shall give you the distance from 6 unto 4, and from 6 unto 8, in your Horizontal Plane. The like may be done for the half hours and quarters.

So also in the Vertical declining Plane. If you first take out the Secant of the declination of the Plane, and prick it down in the Horizontal Line from A unto E, and through E draw right Lines Parallel to the Meridian, which will cut the former hour Lines of 3 and 9, or one of them in the Point C; then take out the Semidiameter A V, and and prick it down in these Parallels from C unto D, and draw right Lines from A unto C, and from V unto D, the Line V D shall be the hour of 6, and if you divide these Lines A C and D C, in such sort as you divided the like Line D C in the Horizontal Plane, you shall have all the hour Points required.

Or you may find the Point D, in the hour of 6, without knowledge either of H or C. For having pricked down A V in the Meridian Line, and A E in the Horizontal Line, and drawn Parallels to the Meridian through the Points at E, you may take the Tangent of the Latitude out of the Sector, and fit it over in the Sines of 90 and 90: so the Parallel Sine of the Declination measured in the same Tangent Line, shall there shew the Complement of the Angle D V A, which the hour Line of 6 maketh with the Meridian; then having the Point D, take out the Se

midiameter V A, and prick it down in thoſe Parallels from D unto C: ſo ſhall you have the Lines D C and A C to be divided as before.

The like might be uſed for the hour Lines upon all other Planes. But I muſt not write all that may be done by the Sector. It may ſuffice that I have wrote ſomething of the Vſe of each Line, and thereby given the ingenuous Reader occaſion to think of more.

The Concluſion to the Reader.

IT is well known to many of you, that this Sector was thus contrived, the moſt part of this Book written in Latine, many Copies tranſcribed and diſperſed more than ſixteen years ſince. I am at the laſt contented to give way that it come forth in Engliſh. Not that I think it worthy either of my labour, or the publick view, but partly to ſatisfie their importunity, who not underſtanding the Latine, yet were at the charge to buy the Inſtrument, and partly for my own eaſe. For as it is painful for others to tranſcribe my Copy, ſo it is troubleſome for me to give ſatisfaction herein to all that deſire it. If I find this to give you content, it ſhall incourage me to do the like for my Croſs-ſtaff, *and ſome other Inſtruments. In the mean time bear with the Printers faults, and ſo I reſt.*

Greſham Coll. 1 Maij. 1623. *E. G.*

FINIS.